Asian Journal of Economics, Finance and Management

Volume 6, Issue 1, Page 132-145, 2024; Article no.AJEFM.1527

Mathematical Modelling of Dynamic Growth of an Infant Financial Market with the Introduction of Technical Awareness

Maryam, M.A [a*], E. O. Ogbaji [a] and M. A. Nwaokolo [a]

[a] Department of Mathematics and Statistics, Federal University Wukari, Nigeria.

Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

Article Information

Open Peer Review History:
This journal follows the Advanced Open Peer Review policy. Identity of the Reviewers, Editor(s) and additional Reviewers, peer review comments, different versions of the manuscript, comments of the editors, etc are available here: https://prh.globalpresshub.com/review-history/1527

Original Research Article

Received: 22/01/2024
Accepted: 25/03/2024
Published: 06/04/2024

ABSTRACT

This research explores the dynamic growth of an infant financial market through Mathematical Modelling, taking into account the influence of technical awareness. The study delves into how various factors, such as market participants' awareness level, and impact the market evolution over time. By developing and analyzing a comprehensive mathematical framework, this research is targeted at enhancing our understanding of how technical awareness affects the growth trajectory of emerging financial market. The result contributes to both financial market theory and practical strategies for market development. The Mathematical Model was formulated based on system of ordinary differential equations to study the growth dynamics of an infant financial market with the introduction of technical awareness. The uniqueness and existence of the model was obtained using Derrick and Grossman theorem, the growth production number of the model was computed using the next generation matrix approach. The investors-free-equilibrium (IFE) state and

**Corresponding author: Email: maryamshamakim@gmail.com;*

Investors' coexistence equilibrium (ICE) state was established. The local stability of the model was also analyzed and it is said to be stable at investor free equilibrium and unstable at the investor coexistence equilibrium. Finally, numerical simulation of the model was carried out and the result shows that an increase in technical investors leads to increase in actual investors and causes decline in quitting investors. That is to say, increase in technical awareness will increase investment rate and decreases quitting investors' rate.

Keywords: financial market; market dynamics; infant market.

1. INTRODUCTION

In recent years, financial market has undergone dynamic shift, reflecting the interconnectedness of global economies and the influence of emerging technologies Giovannetti (2013). The growth trajectory of these market is a subject of significant interest due to its implication on economic stability and investor confidence. The complex interplay between market dynamics and the awareness of market participant has become a critical aspect to consider, giving the information-rich environment in which financial decision are made.

Some studies on modelling the growth dynamics of infant financial market have been done using data driven models. These include autoregressive models studied by Betz [1]. His work focused on Modelling of equities on the market rather than the growth and structure of the market. Networks models by Rice [2] focusing on links between assets and equities on different markets hence exposing loop holes in pricing and lastly computer-networks driven models Giovanni et al. [3] focusing on the factors to consider before choosing an algorithmic model for trading.

Katende [4] study a deterministic compartmental model of three compartments to determine the growth of infant financial market base on the assumption that the rate of growth of the susceptible group should be strictly positive. That is to ensure increase in the number of people with the ability to trade on infant financial market, the rate of change of the size of the investments proportion of the population has to be steadily growing, and the rate of change of the population proportion that quit trading on the infant market should be realized as zero, even though that might take time. The result obtained shows that the impact of single investors on the market is depending on the rate at which people choose to invest and the rate at which people leave the market. It is observed that single investors are leaving financial market due to non-investments related issues which include policy on the market, socio-political and investment environments among others that affect infant financial market [5-9].

Models of physical expansion of investor base on an infant financial market have not been explored so as to understand the key factions of the population to focus on that can grow the number of traders by over whelming proportion [10-14]. In this thesis we aim to explore the mathematical Modelling of the dynamic growth of an infant financial market with particular focus on incorporating the role of awareness in shaping market behavior [15-18]. By integrating concept from mathematical finance and behavioral economics, this research seeks to provide a comprehensive framework for understanding how market growth and participant interact, shading more light on the factors that contribute to the evolving nature of financial systems [19-20]. Through combination of quantitative analysis and Modelling techniques, this study aims to contribute to the broader discourse on market dynamics and provide valuable insights for policy makers, investors, and researcher alike.

2. METHODS

2.1 Model Formulation

1. We assume that potential investors are recruited by the society at a level of λ
2. We assume that the potential investors (S) becomes actual investors at a level (βSI) where (β) is the interaction rate between the potential investors (S) and the actual investors (I). This population increased by the progression of the technical investors at the rate (aA). Also, the population of the potential investors increases due to the coming of the investors whose business collapse and

become potential investors again at the rate (ωI) This class decrease by natural death denoted by (δS)

3. The actual investors (I) are increased by the progression of individuals from the potential investors class (βSI) and by the progression of some individual from the technical investors (bA) and also from the quitting investors at the rate (ηR) which is subject to the acceptance of technical awareness. This class reduces as a result of death rate (δI) and as a result of some actual investor who becomes technical investors (eI) and also as a result of some investors who quit investment at the rate (θI) so also, as a result of investors whose business collapse at the rate (ωI).
4. The quitting investor's class (R) can be increased by the investors that are tired of investment at the rate (θI) and by the technical investors at the rate (αA). The class reduces at the rate (ηR) that is the re-investors which as a result of receiving technical awareness re-invested, death also reduces the class at the rate (δR).
5. The technical investors (A): We assume this is where all the investors receive awareness about all the investment. The population reduces as a result of death at the rate (δA) some becomes potential investors at the rate (aA) others becomes actual investors at (bA) and others move to quitting investors at (αA). While the population increases as a result of actual investors who becomes the technical investors

2.2 Model Assumptions

The Development of the model is based on the following assumptions.

1. We assume that the rate of growth of the potential investors should be strictly positive in other to ensure increase in the number of people with the ability to trade on the infant market.
2. We assume that the rate of change of the size of the investment proportion of the population has to be steadily growing.
3. We assume that the rate of change of the population proportion that quite trading on the infant market should be realized as zero which will take a relatively long time to achieve that.
4. We assume that all the variables and parameters are positive
5. We assume that people can die naturally or as a result of any disasters.

2.3 Modified Model Equations

$$\frac{dS}{dt} = \lambda + \omega I + aA - \beta SI - \delta S \qquad (1.1)$$

$$\frac{dI}{dt} = \beta SI + bA + \eta R - (\theta + e + \delta + \omega)I \qquad (1.2)$$

$$\frac{dR}{dt} = \theta I + \alpha A - (\eta + \delta)R \qquad (1.3)$$

$$\frac{dA}{dt} = eI - (a + b + \alpha + \delta)A \qquad (1.4)$$

2.4 Existence and Uniqueness

Theorem 1: According to Derrick and Grossman [21],

let

$$X_1^1 = f_1(x_1, x_2 \cdots\cdots x_n, t), X_1(t_0) = X_{10}$$
$$X_2^1 = f_2(x_1, x_2 \cdots\cdots x_n, t), x_2(t_0) = X_{20}$$
$$\vdots$$
$$X_n^1 = f_n(x_1 \cdot x_2, \cdots\cdots x_n, t), x_n(t_0) = X_{n0} \qquad (1.5)$$

Let D denote the region in [$(n+1)$ – dimensional space, one dimension for t and n dimensions for the vector X]

If the partial derivatives, $\frac{\partial f_i}{\partial x_i}, i, j = 1,2,3,\ldots,n$ are continuous in

$$D = [(x,t), |t - t_0| \le a, |x - x_0| \le b] \qquad (1.6)$$

Then there is constant $\partial < 0$ such that there exist a unique continuous vector solution

$$X = \{\chi_1(t), \chi_2(t), \ldots\ldots\ldots\ldots \chi_n\} \quad \text{in the interval} \quad |t - t_0| \le \partial \tag{1.7}$$

Theorem 2:

Let:

$$\frac{dS}{dt} = f_1 = \lambda + \omega I + aA - \beta SI - \delta S \quad S(t_0) = S_0 \tag{1.8}$$

$$\frac{dI}{dt} = f_2 = \beta SI + bA + \eta R - (\theta + e + \delta + \omega)I \quad I(t_0) = I_0 \tag{1.9}$$

$$\frac{dR}{dt} = f_3 = \theta I + \alpha A - (\eta + \delta)R \quad R(t_0) = R_0 \tag{1.10}$$

$$\frac{dA}{dt} = f_4 = eI - (a + b + \alpha + \delta)A \quad A(t_0) = A_0 \tag{1.11}$$

$$D = [(S, I, R, A), |S - S_0| \le a, |I - I_0| \le b, |R - R_0| \le c, |A - A_0| \le d.]$$

Then equation $(1.1 - 1.4)$ has a unique solution.

Proof

$$\frac{df_1}{dS}|_{(0,0,0,0)} = |-\beta I - \delta| < \infty$$

$$\frac{df_1}{dI}|_{(0,0,0,0)} = /-\beta S/ < \infty$$

$$\frac{df_1}{dR}|_{(0,0,0,0)} = 0 < \infty$$

$$\frac{df_1}{dA}|_{(0,0,0,0)} = /a/ < \infty$$

$$\frac{df_2}{dS}|_{(0,0,0,0)} = /\beta I/ < \infty$$

$$\frac{df_2}{dI}|_{(0,0,0,0)} = /\beta S - (\theta + e + \delta + \omega)/ < \infty$$

$$\frac{df_2}{dR}|_{(0,0,0,0)} = /\eta R/ < \infty$$

$$\frac{df_2}{dA}|_{(0,0,0,0)} = /b/ < \infty$$

$$\frac{df_3}{dS}|_{(0,0,0,0)} = /0/ < \infty$$

$$\frac{df_3}{dI}|_{(0,0,0,0)} = /\theta/ < \infty$$

$$\frac{df_3}{dR}|_{(0,0,0,0)} = /-(\eta+\delta)/ < \infty$$

$$\frac{df_3}{dA}|_{(0,0,0,0)} = /\alpha/ < \infty$$

$$\frac{df_4}{dS}|_{(0,0,0,0)} = /0/ < \infty$$

$$\frac{df_4}{dI}|_{(0,0,0,0)} = /e/ < \infty$$

$$\frac{df_4}{dR}|_{(0,0,0,0)} = /0/ < \infty$$

$$\frac{df_4}{dA}|_{(0,0,0,0)} = /-(a+b+\alpha+\delta)/ < \infty$$

Therefore, $/\frac{\partial f_i}{\partial xi}/, i, j = 1,2,..........n$ are continuous and bounded.

Hence the equations $(1.1-1.4)$ has a unique solution and the solution exist

2.5 Equilibrium State of the Model

Here we present the investors free equilibrium (IFE) state and the investors' coexistence equilibrium (ICE) state of the model.

2.6 Investors Free Equilibrium (IFE) State

From the model equations: $(1.1)-(1.4)$

At critical point $\frac{dS}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = \frac{dA}{dt} = 0$

Such that: at investors free equilibrium point we have:

$$\left(S_0, I_{0,} R_0 A_0\right) = \left(\frac{\lambda}{\delta}, 0, 0, 0\right) \tag{1.12}$$

2.7 Investors Coexistence Equilibrium (ICE) State

Hence, at investors' coexistence equilibrium (ICE) point we have:

$$S^*, I^*, R^*, A^* = \begin{bmatrix} \frac{MNP - M\eta\theta - Nbe - \eta\alpha e}{MN\beta}, \frac{MNP\delta + \eta\alpha e\delta + \eta M\theta\delta + Nbe\delta + MN\beta\lambda}{MN\beta\omega - N\beta ae - N\beta be - \eta M\theta\beta - \eta\alpha e\beta + MNP\beta}, \\ \frac{(M\theta + \alpha e)[-MNP\delta + \eta\alpha e\delta + \eta M\theta\delta + Nbe\delta + MN\beta\lambda]}{MN(MN\beta\omega - N\beta ae - N\beta be - \eta M\theta\beta - \eta\alpha e\beta + MNP\beta)}, \frac{e(MNP\delta + \eta\alpha e\delta + \eta M\theta\delta + Nbe\delta + MN\beta\lambda)}{M(MN\beta\omega - N\beta ae - N\beta be - \eta M\theta\beta - \eta\alpha e\beta + MNP\beta)} \end{bmatrix} \tag{1.13}$$

2.8 Growth Production Number of the Investors Free Equilibrium

According to Deikman and Heasterbeek (2000), the basic reproduction number is the expected number of secondary infection produced when one infected individual introduced completely into a susceptible population. Based on these contents, the growth production number R_0 is the expected number of investors produced when an investor is introduced completely into the potential investors' population. Computation of R_0 involves product of investment rate and the period or duration of the investment, it determines how long will it takes for a market to expand or grow in a population .If $R_0 < 1$ it means that an investor produces an average less than one investor which by calculation means the market will crash with time ,on the other hand if $R_0 > 1$ it means an investor produces more than one investor throughout his investment period in the market and as such ,the market will invade the population.

Growth production number $\mathrm{R}_0 = \rho(\mathrm{FV})$ where ρ denote spectral radius of matrix V which represent the dominant non-negative Eigen value of the next generation matrix of the square matrices F and V of order $(m \times m)$ where m is the number of investors and is defined by : $F = \frac{\partial f_i}{\partial x_j}$ and $V = \frac{\partial V_i}{\partial x_j}$

Such that; V is a non-singular m matrix.

Considering the equations $(1.1-1.4)$ the first equation are not investors but they can be through influence and contact with the investors, the second equation are the investors, the third are the investors that quite investment and the fourth are technical that can spread the business that means enlightens team. So taking the second and fourth equations in order to evaluate the growth production number,

$$F = \beta SI \tag{1.13}$$

We let $f_{i,j} = \beta SI$

$g_{i,j} = 0$

$$F = \begin{pmatrix} \frac{\beta\lambda}{\delta} & 0 \\ 0 & 0 \end{pmatrix} \tag{1.14}$$

Also $$V = \begin{pmatrix} bA + \eta R - PI \\ eI - MA \end{pmatrix} \tag{1.15}$$

Let $f_{i,j} = bA - \eta R - PI$

$g_{i,j} = eI - MA$

$$V = \begin{pmatrix} P & -b \\ -e & M \end{pmatrix} \tag{1.16}$$

$$V^{-1}=\begin{pmatrix}\frac{M}{MP-be} & \frac{b}{MP-be}\\ \frac{e}{MP-be} & \frac{P}{MP-be}\end{pmatrix}$$

$$FV^{-1}=\begin{pmatrix}\frac{\beta\lambda}{\delta} & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}\frac{M}{MP-be} & \frac{b}{MP-be}\\ \frac{e}{MP-be} & \frac{P}{MP-be}\end{pmatrix}$$

$$FV^{-1}=\begin{pmatrix}\frac{\beta\lambda M}{\delta(MP-be)} & \frac{\beta\lambda b}{\delta(MP-be)}\\ 0 & 0\end{pmatrix} \tag{1.17}$$

Now we compute $\left|FV^{-1}-I\tau\right|=0$ where

$\tau=$ Eigen values and $I=$ identity matrix for (2×2)

$R_0=\rho\left(FV^{-1}\right)$ Which means the dominant Eigen value of $\left(FV^{-1}\right)$. $\rho=Rho$ which is the spectral radius.

Computation of $\left|FV^{-1}-I\tau\right|=0$, gives:

$$\begin{pmatrix}\frac{\beta\lambda M}{\delta(MP-be)}-\tau_1 & \frac{\beta\lambda b}{\delta(MP-be)}\\ 0 & -\tau_2\end{pmatrix}=0$$

Now, $\left(\frac{\beta\lambda M}{\delta(MP-be)}-\tau_1\right)\tau_2=0$

$$\frac{\beta\lambda M}{\delta(MP-be)}-\tau_1=0$$

$$\tau_2=0 \tag{1.18}$$

$$\therefore \tau_1=\frac{\beta\lambda M}{\delta(MP-be)}$$

2.9 Local Stability of the Model

2.9.1 Local stability of the investors free equilibrium

To investigate the local stability of the investors free equilibrium points of the model, we linearizes the model by computing its Jacobian matrix J given by:

$$\Rightarrow J=\begin{pmatrix} -\delta & \omega-\beta\frac{\lambda}{\delta} & 0 & a \\ 0 & \beta\frac{\lambda}{\delta}-P & \eta & b \\ 0 & \theta & -N & \alpha \\ 0 & e & 0 & -M \end{pmatrix} \tag{1.19}$$

Now we find the characteristics equation which is given by Det.$(A-I\tau)=0$ Where τ is the eigen value and A is an $n\times n$ matrix. Hence, we replace the $n\times n$ matrix (A) by the jacobian matrix (J), Thus:

$$\Rightarrow\begin{pmatrix} -\delta-\tau & \frac{\omega\delta-\beta\lambda}{\delta} & 0 & a \\ 0 & \frac{\beta\lambda-P\delta}{\delta}-\tau & \eta & b \\ 0 & \theta & -N-\tau & \alpha \\ 0 & e & 0 & -M-\tau \end{pmatrix}$$

We let $\left(\frac{\beta\lambda-P\delta}{\delta}\right)=Q$ such that

$$(-\delta-\tau)\left[Q-\tau\{MN+M\tau+N\tau+\tau^2\}-\eta\{-M\theta-\tau\theta\}+b\{eN+\tau e\}\right]=0$$
$$\Rightarrow(-\delta-\tau)\left[MNQ-MN\tau+MQ\tau-M\tau^2+NQ\tau-N\tau^2+Q\tau^2-\tau^3+M\eta\theta+\eta\theta\tau+Nbe+be\tau\right]=0$$
$$\Rightarrow(-\delta-\tau)=0$$
$$-\delta=\tau_1, \tag{1.20}$$

$$\left(MNQ+M\eta\theta+Nbe-MN\tau+MQ\tau+NQ\tau+\eta\theta\tau+be\tau-M\tau^2-N\tau^2+Q\tau^2-\tau^3\right)=0$$

$$\tau^3+\tau^2(Q-N-M)+\tau(be+\eta\theta+NQ+MQ-MN)+MNQ+M\eta\theta+Nbe=0$$

We assume all the coefficient of (τ) to be constant ≥ 0 so that

$$\tau_2^3+\tau_3^2+\tau_4+1=0$$
$$\tau^2(\tau+1)+1(\tau+1)=0$$
$$(\tau^2+1)(\tau+1)=0$$
$$\tau+1=0$$
$$\tau_2=-1 \tag{1.21}$$
$$\tau^2+1=0$$
$$\tau^2=-1\Rightarrow\tau_3\tau_4=\sqrt{-1} \tag{1.22}$$

Hence the model is stable as the value of (τ) is negative. According to Routh-Hurwitz theorem which stated that an equilibrium state will be asymptotically stable if and only if all the Eigen values of the characteristics equation of the matrix $(A-I\tau)=0$ have negative real part. Since we have two negatives real eigen values and two imaginaries, we conclude that the Investor Free Equilibrium (IFE) state of the model is asymptotically stable, this becomes unstable if the Eigen values is greater than zero or positive. But we have

$\tau_1 = -\delta$, $\tau_2 = -1$ $\tau_3 \tau_4 = -i$

2.9.2 Local stability at investors coexistence

The long-time dynamics of an infant financial market is characterized by the stability at the coexistence of investor equilibrium. In order to determine the dynamics of an infant financial market, we investigate the stability of the model at the investor coexistence equilibrium (ICE) The jacobian of the system is

$$J^* = \begin{pmatrix} -\beta\left(-\frac{MNP\delta + \eta\alpha e\delta + \eta M\theta\delta + Nbe\delta + MN\beta\lambda}{MN\beta\omega - N\beta ae - N\beta be - \eta M\theta\beta - \eta\alpha e\beta + MNP\beta}\right) - \delta & \omega - \beta\left(\frac{MNP - M\eta\theta - Nbe - \eta\alpha e}{MN\beta}\right) & 0 & a \\ \beta\left(-\frac{MNP\delta + \eta\alpha e\delta + \eta M\theta\delta + Nbe\delta + MN\beta\lambda}{MN\beta\omega - N\beta ae - N\beta be - \eta M\theta\beta - \eta\alpha e\beta + MNP\beta}\right) & \beta\frac{MNP - M\eta\theta - Nbe - \eta\alpha e}{MN\beta} - P & \eta & b \\ 0 & \theta & -N & \alpha \\ 0 & e & 0 & -M \end{pmatrix} \quad (1.23)$$

Now we find the characteristic equation which is given by Det. $(A - I\tau) = 0$ Where τ is the eigen value and A is an $m \times m$ matrix. We replace the $m \times m$ matrix A by the jacobian matrix (J^*), Thus;

Det. $(J^* - I\tau) =$

$$\begin{pmatrix} -\beta\left(-\frac{MNP\delta + \eta\alpha e\delta + \eta M\theta\delta + Nbe\delta + MN\beta\lambda}{MN\beta\omega - N\beta ae - N\beta be - \eta M\theta\beta - \eta\alpha e\beta + MNP\beta}\right) - \delta - \tau_1 & \omega - \beta\left(\frac{MNP - M\eta\theta - Nbe - \eta\alpha e}{MN\beta}\right) & 0 & a \\ \beta\left(-\frac{MNP\delta + \eta\alpha e\delta + \eta M\theta\delta + Nbe\delta + MN\beta\lambda}{MN\beta\omega - N\beta ae - N\beta be - \eta M\theta\beta - \eta\alpha e\beta + MNP\beta}\right) & \beta\frac{MNP - M\eta\theta - Nbe - \eta\alpha e}{MN\beta} - P - \tau_2 & \eta & b \\ 0 & \theta & -N - \tau_3 & \alpha \\ 0 & e & 0 & -M - \tau_4 \end{pmatrix}$$

$$-\beta\left(-\frac{MNP\delta + \eta\alpha e\delta + \eta M\theta\delta + Nbe\delta + MN\beta\lambda}{MN\beta\omega - N\beta ae - N\beta be - \eta M\theta\beta - \eta\alpha e\beta + MNP\beta}\right) - \delta - \tau_1 \begin{bmatrix} \frac{MNP - M\eta\theta - Nbe - \eta\alpha e}{MN} - P - \tau_2 & \eta & b \\ \theta & -N - \tau_3 & \alpha \\ e & 0 & -M - \tau_4 \end{bmatrix} = 0$$

$$\left(\frac{MNP\delta - \eta\alpha e\delta - \eta M\theta\delta - Nbe\delta - MN\beta\lambda}{MN\omega - Nae - Nbe - \eta M\theta - \eta\alpha e + MNP}\right) - \delta - \tau_1\left(\frac{MNP - M\eta\theta - Nbe - \eta\alpha e}{MN} - P - \tau_2\right)\begin{bmatrix} -N - \tau_3 & \alpha \\ 0 & -M - \tau_4 \end{bmatrix} = 0$$

$$\left(\frac{MNP\delta - \eta\alpha e\delta - \eta M\theta\delta - Nbe\delta - MN\beta\lambda}{MN\omega - Nae - Nbe - \eta M\theta - \eta\alpha e + MNP}\right) - \delta - \tau_1\left(\frac{MNP - M\eta\theta - Nbe - \eta\alpha e}{MN} - P - \tau_2\right) = 0$$

$$\begin{bmatrix} -N - \tau_3 & \alpha \\ 0 & -M - \tau_4 \end{bmatrix} = 0$$

$$(-N - \tau_3)(-M - \tau_4) = 0$$

$$\left(\frac{MNP\delta - \eta\alpha e\delta - \eta M\theta\delta - Nbe\delta - MN\beta\lambda}{MN\omega - Nae - Nbe - \eta M\theta - \eta\alpha e + MNP}\right) - \delta - \tau_1 = 0$$

$$\left(\frac{MNP - M\eta\theta - Nbe - \eta\alpha e}{MN} - P - \tau_2\right) = 0$$

$\left(-N - \tau_3\right) = 0$ And

$\left(-M - \tau_4\right) = 0$

So that the Eigen values are:

$$\tau_1 = \frac{MNP\delta - \eta\alpha e\delta - \eta M\theta\delta - Nbe\delta - MN\beta\lambda}{MN\omega - Nae - Nbe - \eta M\theta - \eta\alpha e + MNP} - \delta \qquad (1.24)$$

$$\tau_2 = \frac{MNP - M\eta\theta - Nbe - \eta\alpha e}{MN} - P \qquad (1.25)$$

$$\tau_3 = -N \qquad (1.26)$$

$$\tau_4 = -M \qquad (1.27)$$

The model is unstable as we have two negative and two positive value of τ using the same theorem as above we therefore, conclude that the investor coexistence of the model is unstable as it does not satisfy the theorem.

3. COMPUTATIONAL RESULT

In this section we carried out the numerical simulations of the financial market investment model with the parameters value in Table 1, considering the effect of technical awareness on financial market. Varying the model parameters in order to analyzed the sensitivity of each parameter. All model graphs are generated from the trend of the population after the code was implemented on maple18 software, parameter values represented in Table 1 produces Fig.1. to Fig. 4.

4. DISCUSSION OF RESULT

In this section we discussed both the analytical and computational solutions of the extended model.

4.1 Analytical Results

4.1.1 Model extension

The model by Katende [4] was extended from three compartments to four compartments by incorporating market technical awareness compartments (*A*=Technical investor compartment*)* with four different rates. The contact rate of potential investor with technical investor, the contact rate of actual investor with the technical investor, the contact rate of quitting investor with technical investor, and the rate at which actual investor becomes technical investors.

4.1.2 Existence and uniqueness

The extended model was tested for existence and uniqueness using Derrick and Grossman theorem of [5] and found out that the model solution exists and has a unique solution.

4.1.3 Equilibrium state

The extended model equation was solved to obtained two equilibrium state, that is the investor free equilibrium state and the investor coexistence equilibrium state.

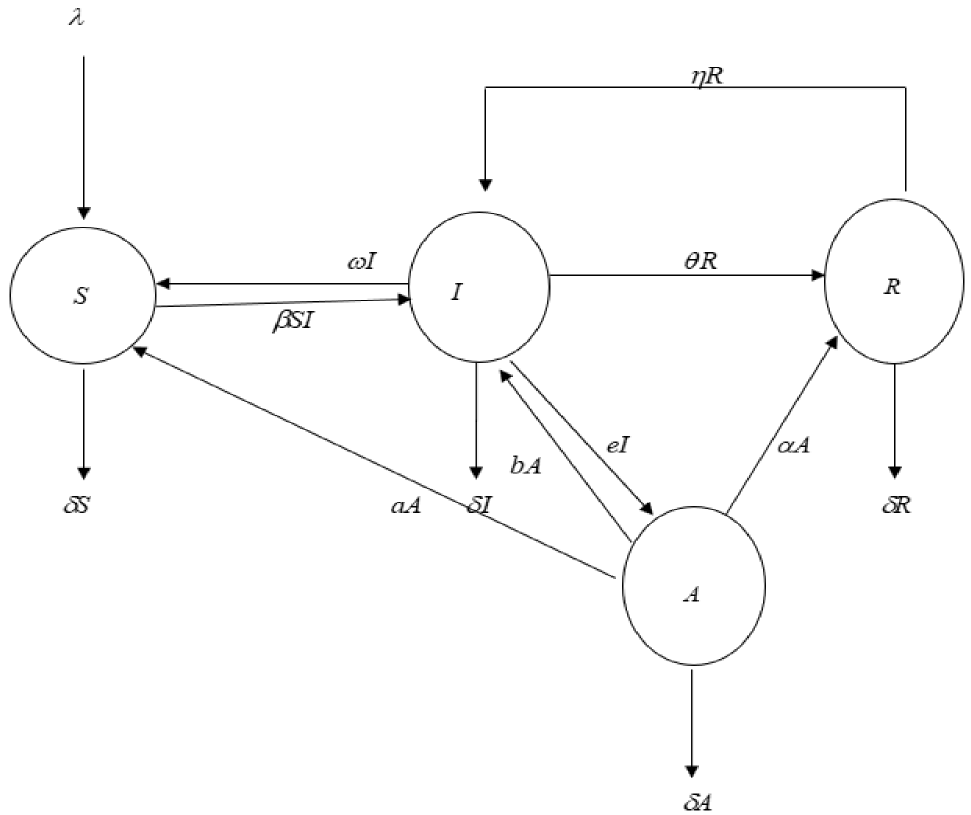

Fig. 1. Schematic Diagram of the Modified Model

Table 1. Model variables and parameters

Variables and Parameters	Descriptions
S	Potential investors
I	Actual investors
R	Quitting investors
A	Technical investors
βSI	Contact rate of the investors
λ	The recruitments rate
θI	Contact rate of quitting investors
ηR	The re-investors rate
ωI	The rate at which actual investor becomes intended investors
∂S	Death rate of potential investors
∂I	Death rate of actual investors
∂R	Death rate of quitting investors
∂A	Death rate of technical investors
aA	Contact rate of potential investors
eI	The contact rate of technical investors
bA	The rate at which technical becomes potential investors.
αA	Contact rate of technical with quitting investors.

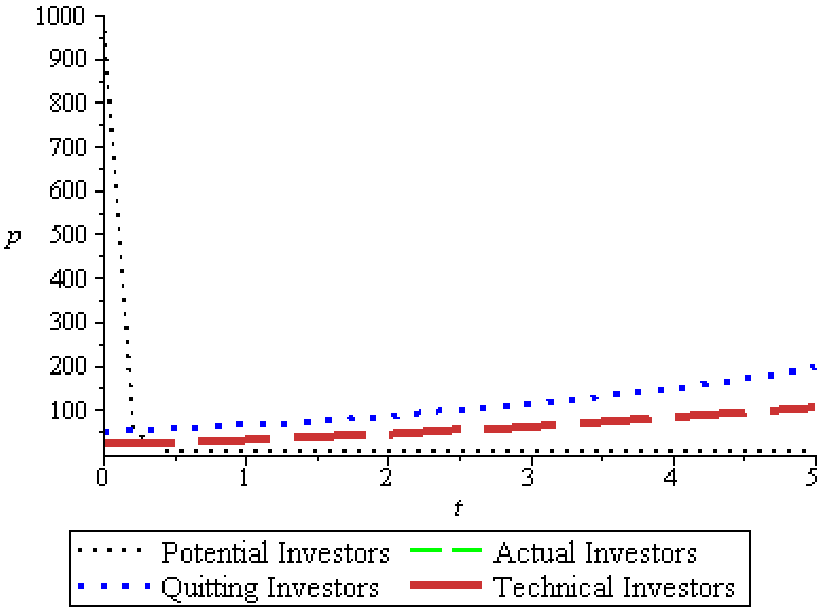

Fig. 2. Model graph 1

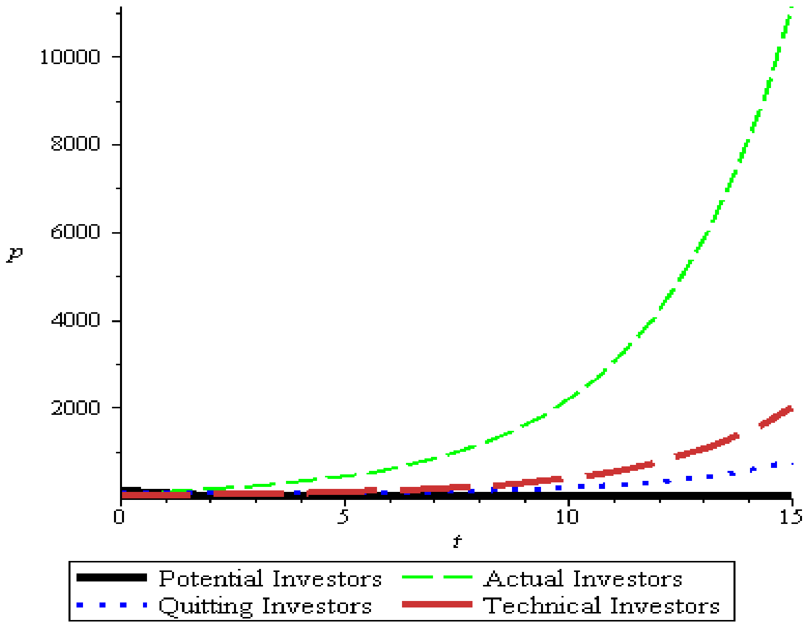

Fig. 3. Model graph 2

4.1.4 Local stability

The local stability of the extended model was computed by linearizing the model equation and computing Jacobian matrix both at investor free equilibrium and at investor coexistence equilibrium. The result shows that, at Investor Free Equilibrium state is locally asymptotically stable and unstable at the Investor Coexistence Equilibrium state using the Routh-Hurwitz theorem.

4.1.5 Growth production number

The Growth production number of the extended model at the investor free equilibrium was obtained using the next generation matrix and the result was found to be less than one.

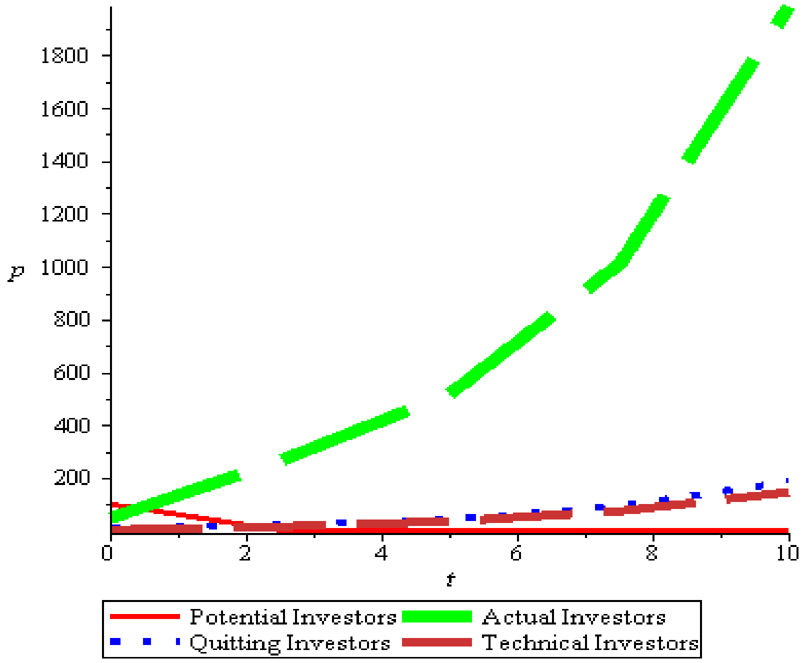

Fig. 4. Model graph 3

4.1.6 Numerical simulation

The numerical computation was considered in two categories; when the level of awareness is reduced what will happen in the market and when the level increased. In the Figs. (1-4) results both shows the significant effect of awareness in the financial market. The actual investor population reaches a stable stage where it remain constant. In Fig.1 we observe that there no investment as a result of no awareness, increase in actual investor increases quitting investors. Fig.2 shows that increase in technical awareness in order to motivate the investors so as to avoid the quitting of investor increase the number of actual investor to a certain level. In Fig. 4 the investment rate increases to almost extinction when awareness increases and the population of the quitting investors decreases, these has shown the significant effects of awareness in the growth of financial market investments.

5. CONCLUSION

base on the results of this study, we conclude that the most effective way of moderating the dynamic growth of an infant financial market so as to avoid the quitting investors, is to provide adequate awareness in form of enlightenment campaign on radio, television, newspapers, in churches, in masjid and even in schools on the needs for individuals to invest in financial market in order to boast the country economy.

6. RECOMMENDATIONS

Due to economy inflation crisis in the globe, a financial market has a significant impact in economy stability. So therefore, we recommend technical awareness should be re-emphasized to avoid the quitting of investors. We also recommend Government, Local and International organizations who are working very hard to improve and promote infant financial market to come up with economic policies that will encourage people to invest and likewise, create a conducive investment environment and social economic facilities in the market. However, we recommend the work for further studies.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Betz F. Mathematical Modelling of Growth Dynamics of Infant Financial Markets. Journal of Mathematical Finance. 2016; 10(3).

2. Rice O. A Network Model for Financial Markets. PhD Thesis, University College London, South Africa. Journal of Governance and Regulation. 2015;4:154-163.
3. Giovannini A, Iacopetta M, Minetti R. Financial Markets, Banks, and Growth: Disentangling the Links. International Journal of Business and Management. 2011;6:87-104.
4. Katende R. Mathematical modelling of Growth Dynamics of infant Financial Markets. Journal of Mathematic finance. 2020;10:388-398
5. Amadi IU, Wobo GO. A mathematical model analysis for estimating stock market price changes. International Journal of Applied Science and Mathematical Theory. 2022;8(2):38-50.
ISSN 2489-009X P-ISSN 2695-1908.
6. Amaral LAN, Gopikrishnan P, Plerou V, Stanley HE. A model for the growth dynamics of economic organizations Physica A: Statistical Mechanics and its Applications. 2001;299(1):127-136
7. Andrea. FIve way of Mode Serve Society: A manifest. Nature. 2020;58(7813):482-483.
DOI:10.1038/d41586-020-01812-9
hdi:1885/21903.
8. Atia C. Building financial literacy society for development. Camfed model. Accra; 2012.
9. Bosworth BP, Collin SM, Reinhart CM. Capital Flows to Developing Economies: Implications for Saving and Investment. Journal of Mathematical Finance Brookings Institution Press. 1999;1:143-180.
10. Aanjan PG, Nirmalbabu D. A Study on the Investors Awareness in the Equity Market with Special Reference to Coimbatore City. International Journal of Creative Research Thought (IJCRT) 2021;10.
ISSN 2320-2882.
11. Adesanya BM, Adediji AM, Okenna NP. Stock exchange market activities and economic development: Evidence from the Nigerian Economy. Bingham Journal of Economics and Allied Studies (BJEAS). 2020;4(2):231-345.
12. Alex B, Irina ME., Andrey K, Irina MY, Anzhela N. Mathematical modelling for financial analysis of a enterprise: Motivating of Not open innovation. Journal of Open Innovation; Technology, Market and Complexity. Basel. 2021;7 (1):1-15.
ISSN 2199-8531, MDPI
13. Alfaro L, Chanda A., Kalemli-Ozcan, S, Sayek S. Does foreign direct investment promote growth? Exploring the role of financial markets on linkages. Journal of Development Economics. 2010;91(2):242-256
14. Carraro F. (2019) Language of the Emerging Financial Market. Essays in Economic & Business Development: A Century of Competition between French Banks and Notaries. Explorations in Economic History. *Economics World,* 5, 268-280.
15. Dheeraj V. 2023) Financial Modelling Guide. *Article by Wall Street Mojo Teem.*
16. Diekman, O & Heesterbeck, J. A. P. (2000) Mathematical epidemiology of infectious disease. Wiley Series in Mathematical and Computational Biology. John Wiley & Sons, West Sussex, England, 2000
17. Donald G, Maxwell S. The Determinant of Stock Market Participation: Evidence from individual Investors in Zimbabwe. Acta University Danubius. 2018;14(4):
18. Eleanor Kurby and Shane Worner (SWP3/2014) Crowd funding: An infant industry Growing fast. Wall Street Editorial Team Reviewed by Dheeraj Vaidya, CFA, ERM
19. Emerson R, Hall SG, Zalewska-Mitura A. Evolving market efficiency with an european Central Bank. (2001) The Role of Financial Markets for Economic Growth. European Economic Review, 1997;37:613-622.
20. Fenglian W, Chia-Huei W, Sang-Bing T. Research article. A journal of mathematical problems in engine; 2020.
21. Derrick NR, Grossman SL. Differential Equation with application. Addison Wesley publishing company, Inc. Philippiness; 1976.

Peer-review history:
The peer review history for this paper can be accessed here:
https://prh.globalpresshub.com/review-history/1527