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Complexiton Solutions of Nonlinear Partial Differential Equations Using a New Auxiliary Equation

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Abstract

In this paper, a novel auxiliary equation: $\varphi'' = a + b\varphi + c\varphi^3$ which has mutiple function solutions including trigonometric function, hyperbolic function and other functions, is considered. It is applied to a series of partial differential equations easily and effectively. It helps physicists to obtain complexiton solutions of nonlinear partial equations and analyze special phenomena accurately in their fields.

Keywords: Complexiton solution; Riccati equation; Partial differential equation

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1 Introduction

As it is well-known, many physics and nature science are usually characterized well by ubiquitous nonlinear dynamical equations. Soliton theory is one of significant fields in nonlinearity, travelling wave solutions of mathematical and physics nonlinear models especially the most representative equations like the KdV equation [1] and Hirota-Satsuma equations are very important, they have been committed to assist people in describing the nature science better. The KdV equation is derived by Korteweg and de Vries to model the evolution of shallow water wave in 1895. The (2+1) dimensional KdV equation manifest the change of shallow water wave accurately, it is obtained through potential function. Hirota-Satsuma equations are classified as a soliton equation by B. Fuchssteiner, it has a bi-hamiltonian formulation and obtains countably many conserved quantities and symmetry generators. Complete integrability of this equation is conjectured by Hirota and Satsuma [2].

In previously, the complexiton solutions (interaction solutions) show that interaction between different kinds of travelling wave solutions of nonlinear evolution equations, they attracted numerous attention, They are usually tended to uncovering potential meaningful applications.

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In currently, multifarious methods for searching analytical solutions of nonlinear equations have evolved from complicated process with heavy computation to simpleness and understandability. Sub-equation approach which contains the the homogeneous balance method [3], sine-cosine method [4], the sech-function method [5], the hyperbolic tangent function method [6, 7], the multiple exp-function method [8, 9], the Riccati equations method [10] as widespread application to construct exact solutions. In 2008, the G'/G-expansion method was proposed by Wang [11] which arose a large of attention as its straightforward, simplification and applicability in obtaining analytical solutions of nonlinear equations. Subsequently authors developed the the G'/G-expansion method to improved G'/G method and extended G'/G methods. It has been successfully to get rational solutions, trigonometric and hyperbolic function solutions of many kinds of nonlinear evolution equations [12, 13, 14, 15, 16, 17, 18, 19] through the auxiliary equation $G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0$, where λ, μ are arbitrary constants. But the solutions of the solvable auxiliary equation are singular soliton solutions which are not contain complexiton solutions in applying the basis G'/G-expansion and the other G'/G-expansion method. In this paper, the novel sub-equation can obtain interaction solutions successfully.

Ma [20], Fan [21], Chen [22, 23], Chen [24, 25, 26, 27, 28], Yan [29, 30] devoted to constructing special soliton solutions by using combination of auxiliary equations and got great success. In this paper, a new sub-equation: $\varphi'' = a + b\varphi + c\varphi^3$ which has mutiple function solutions including trigonometric function, hyperbolic function and other functions.

2 New Solutions of the Auxiliary Equation

The desired equation reads:

$$\varphi'' = a + b\varphi + c\varphi^3, \tag{1}$$

where $\varphi'' = \varphi''(\xi)$. In order to work out $\varphi(\xi)$, hypothesis are taken as follow:

$$\varphi(\xi) = a_0 + \frac{a_1 F(\xi)H(\xi) + a_2 G'(\xi)H'(\xi)}{a_3 F(\xi) + 1}, \tag{2}$$

where $F(\xi), G(\xi), H(\xi)$ are functions satisfying the following Riccati equations respectively. In addition a_0, a_1, a_2, a_3 are constants to be determined later.

$$F'(\xi) = A_1 + B_1 F(\xi) + C_1 F^2(\xi)$$
$$G'(\xi) = A_2 + B_2 G(\xi) + C_2 G^2(\xi)$$
$$H'(\xi) = A_3 + B_3 H(\xi) + C_3 H^2(\xi), \tag{3}$$

where $A_1, A_2, A_3, B_1, B_2, B_3, C_1, C_2, C_3,$ are arbitrary constants.

Inserting eqno(2) into eqno(1) with the related Riccati auxiliary equations eqno(3), then setting the coefficients of $F^i(\xi)G^j(\xi)H^s(\xi)$ $(0 \le i, j, s \le 6)$ equate to zero, we derive a system of over determined linear equations with a_0, a_1, a_2, a_3 $A_1, A_2, A_3, B_1, B_2, B_3, C_1, C_2, C_3$.

The precondition relationship:

$A_1 = A_1, B_1 = B_1, C_1 = -a_3^2 A_1 + a_3 B_1, A_2 = A_2, B_2 = B_2, C_2 = C_2, A_3 = 0, B_3 = -\frac{B_1}{2} + a_3 A_1, C_3 = 0,$ $a = -\frac{(B_1^2 - 4a_3 B_1 A_1 + 4a_3^2 A_1^2)}{4} a_0, b = \frac{B_1^2 - a_3 B_1 A_1 + a_3^2 A_1^2}{4}, c = 0$ $a_0 = a_0, a_1 = a_1, a_2 = 0, a_3 = a_3.$

Type 1: When $A_1 = \frac{1}{2}, B_1 = 0, C_1 = \frac{1}{2}$

$$\varphi_1 = a_0 + \frac{a_1(\tan(\xi) \pm \sec(\xi))(\sinh(B_3\xi) + \cosh(B_3\xi))}{a_3(\tan(\xi) \pm \sec(\xi)) + 1} \tag{4}$$

$$\varphi_2 = a_0 + \frac{a_1(\csc(\xi) - \cot(\xi))(\sinh(B_3\xi) + \cosh(B_3\xi))}{a_3(\csc(\xi) - \cot(\xi)) + 1} \tag{5}$$

$$\varphi_3 = a_0 + \frac{a_1(\frac{\tan(\xi)}{1\pm\sec(\xi)})(\sinh(B_3\xi)+\cosh(B_3\xi))}{a_3(\frac{\tan(\xi)}{1\pm\sec(\xi)})+1} \tag{6}$$

Type 2: When $A_1 = -\frac{1}{2}, B_1 = 0, C_1 = -\frac{1}{2}$

$$\varphi_4 = a_0 + \frac{a_1(\cot(\xi)\pm\csc(\xi))(\sinh(B_3\xi)+\cosh(B_3\xi))}{a_3(\cot(\xi)\pm\csc(\xi))+1} \tag{7}$$

$$\varphi_5 = a_0 + \frac{a_1(\sec(\xi)-\tan(\xi))(\sinh(B_3\xi)+\cosh(B_3\xi))}{a_3(\sec(\xi)-\tan(\xi))+1} \tag{8}$$

$$\varphi_6 = a_0 + \frac{a_1(\frac{\cot(\xi)}{1\pm\csc(\xi)})(\sinh(B_3\xi)+\cosh(B_3\xi))}{a_3(\frac{\cot(\xi)}{1\pm\csc(\xi)})+1} \tag{9}$$

Type 3: When $A_1 = 1, B_1 = 0, C_1 = 4$

$$\varphi_7 = a_0 + \frac{a_1(\frac{\tan(\xi)}{1-\tan^2(\xi)})(\sinh(B_3\xi)+\cosh(B_3\xi))}{a_3(\frac{\tan(\xi)}{1-\tan^2(\xi)})+1} \tag{10}$$

Type 4: When $A_1 = -1, B_1 = 0, C_1 = -4$

$$\varphi_8 = a_0 + \frac{a_1(\frac{\tan(\xi)}{1-\cot^2(\xi)})(\sinh(B_3\xi)+\cosh(B_3\xi))}{a_3(\frac{\tan(\xi)}{1-\cot^2(\xi)})+1} \qquad (11),$$

where a_0, a_2, a_3, B_2, B_3 are arbitrary constants. Typical explicit solutions are taken into consideration above, and there exists abundant interaction solutions of eqno(1) we omit in the paper. In the wake of such work, we aim at wielding the auxiliary equation to the evolution equations as follows.

3 Applications of this Sub-equation

Example 1 Consider the (2+1) dimensional KdV equation[31].

$$\begin{gathered} U_t + U_{xxx} - 3U_xV - 3UV_x = 0 \\ U_x - V_y = 0, \end{gathered} \tag{12}$$

here, to start off, we have the hypothesis in the following terms are obtained:

$$U(\xi) = \sum_{p=0}^{m} m_p\varphi^p(\xi),$$

$$V(\xi) = \sum_{q=0}^{n} n_q\varphi^q(\xi), \varphi(\xi) = x + ky - vt, \tag{13}$$

here k, v are constants, v represent as the wave speed. Where m, n are positive integers and equate to 2 respectively which are determined by the principle of homogeneous balance. $\varphi(\xi)$ satisfies the sub-equation: $\varphi'' = a + b\varphi + c\varphi^3$.

$$\begin{gathered} U(\xi) = m_0 + m_1\varphi(\xi) + m_2\varphi^2(\xi), \\ V(\xi) = n_0 + n_1\varphi(\xi) + n_2\varphi^2(\xi) \end{gathered} \tag{14}$$

$m_0, m_1, m_2, n_0, n_1, n_2$ are all obtained in the later. Hence, when we substitute eqno(14) into eqno(13) along with aid of auxiliary equation. Equating the coefficients of $\varphi^\alpha(\xi)\varphi'(\xi)$ $(0 \le \alpha \le 3)$ to zero, a set of algebraic equations are yielded that unknown parameters $m_0, m_1, m_2, n_0, n_1, n_2, k, c, v$ are able to solve through using the computation of Maple.

Then analytical interaction solutions of system eqno(12): when $k = k, v = v, m_0 = \frac{1}{3}k(-v+4b-3n_0), m_1 = 0, m_2 = n_2, n_0 = n_0, n_1 = 0, n_2 = n_2$ with type1, type2, type3, type4 as explicit solutions of eqno(1):

$$U_1 = \frac{1}{3}k(-v+4b-3n_0) + n_2(a_0 + \frac{a_1(\tan(\xi) \pm \sec(\xi))(\sinh(B_3\xi)+\cosh(B_3\xi))}{a_3(\tan(\xi) \pm \sec(\xi))+1})^2$$

$$V_1 = n_0 + n_2(a_0 + \frac{a_1(\tan(\xi) \pm \sec(\xi))(\sinh(B_3\xi)+\cosh(B_3\xi))}{a_3(\tan(\xi) \pm \sec(\xi))+1})^2 \qquad (15)$$

$$U_2 = \frac{1}{3}k(-v+4b-3n_0) + n_2(a_0 + \frac{a_1(\csc(\xi) - \cot(\xi))(\sinh(B_3\xi)+\cosh(B_3\xi))}{a_3(\csc(\xi) - \cot(\xi))+1})^2$$

$$V_2 = n_0 + n_2(a_0 + \frac{a_1(\csc(\xi) - \cot(\xi))(\sinh(B_3\xi)+\cosh(B_3\xi))}{a_3(\csc(\xi) - \cot(\xi))+1})^2 \qquad (16)$$

$$U_3 = \frac{1}{3}k(-v+4b-3n_0) + n_2(a_0 + \frac{a_1(\frac{\tan(\xi)}{1\pm\sec(\xi)})(\sinh(B_3\xi)+\cosh(B_3\xi))}{a_3(\frac{\tan(\xi)}{1\pm\sec(\xi)})+1})^2$$

$$V_3 = n_0 + n_2(a_0 + \frac{a_1(\frac{\tan(\xi)}{1\pm\sec(\xi)})(\sinh(B_3\xi)+\cosh(B_3\xi))}{a_3(\frac{\tan(\xi)}{1\pm\sec(\xi)})+1})^2 \qquad (17)$$

$$U_4 = \frac{1}{3}k(-v+4b-3n_0) + n_2(a_0 + \frac{a_1(\cot(\xi) \pm \csc(\xi))(\sinh(B_3\xi)+\cosh(B_3\xi))}{a_3(\cot(\xi) \pm \csc(\xi))+1})^2$$

$$V_4 = n_0 + n_2(a_0 + \frac{a_1(\cot(\xi) \pm \csc(\xi))(\sinh(B_3\xi)+\cosh(B_3\xi))}{a_3(\cot(\xi) \pm \csc(\xi))+1})^2 \qquad (18)$$

$$U_5 = \frac{1}{3}k(-v+4b-3n_0) + n_2(a_0 + \frac{a_1(\sec(\xi) - \tan(\xi))(\sinh(B_3\xi)+\cosh(B_3\xi))}{a_3(\sec(\xi) - \tan(\xi))+1})^2$$

$$V_5 = n_0 + n_2(a_0 + \frac{a_1(\sec(\xi) - \tan(\xi))(\sinh(B_3\xi)+\cosh(B_3\xi))}{a_3(\sec(\xi) - \tan(\xi))+1})^2 \qquad (19)$$

$$U_6 = \frac{1}{3}k(-v+4b-3n_0) + n_2(a_0 + \frac{a_1(\frac{\cot(\xi)}{1\pm\csc(\xi)})(\sinh(B_3\xi)+\cosh(B_3\xi))}{a_3(\frac{\cot(\xi)}{1\pm\csc(\xi)})+1})^2$$

$$V_6 = n_0 + n_2(a_0 + \frac{a_1(\frac{\cot(\xi)}{1\pm\csc(\xi)})(\sinh(B_3\xi)+\cosh(B_3\xi))}{a_3(\frac{\cot(\xi)}{1\pm\csc(\xi)})+1})^2 \qquad (20)$$

$$U_7 = \frac{1}{3}k(-v+4b-3n_0) + n_2(a_0 + \frac{a_1(\frac{\tan(\xi)}{1-\tan^2(\xi)})(\sinh(B_3\xi)+\cosh(B_3\xi))}{a_3(\frac{\tan(\xi)}{1-\tan^2(\xi)})+1})^2$$

$$V_7 = n_0 + n_2(a_0 + \frac{a_1(\frac{\tan(\xi)}{1-\tan^2(\xi)})(\sinh(B_3\xi)+\cosh(B_3\xi))}{a_3(\frac{\tan(\xi)}{1-\tan^2(\xi)})+1})^2 \qquad (21)$$

$$U_8 = \frac{1}{3}k(-v+4b-3n_0) + n_2(a_0 + \frac{a_1(\frac{\tan(\xi)}{1-\cot^2(\xi)})(\sinh(B_3\xi)+\cosh(B_3\xi))}{a_3(\frac{\tan(\xi)}{1-\cot^2(\xi)})+1})^2$$

$$V_8 = n_0 + n_2(a_0 + \frac{a_1(\frac{\tan(\xi)}{1-\cot^2(\xi)})(\sinh(B_3\xi)+\cosh(B_3\xi))}{a_3(\frac{\tan(\xi)}{1-\cot^2(\xi)})+1})^2, \qquad (22)$$

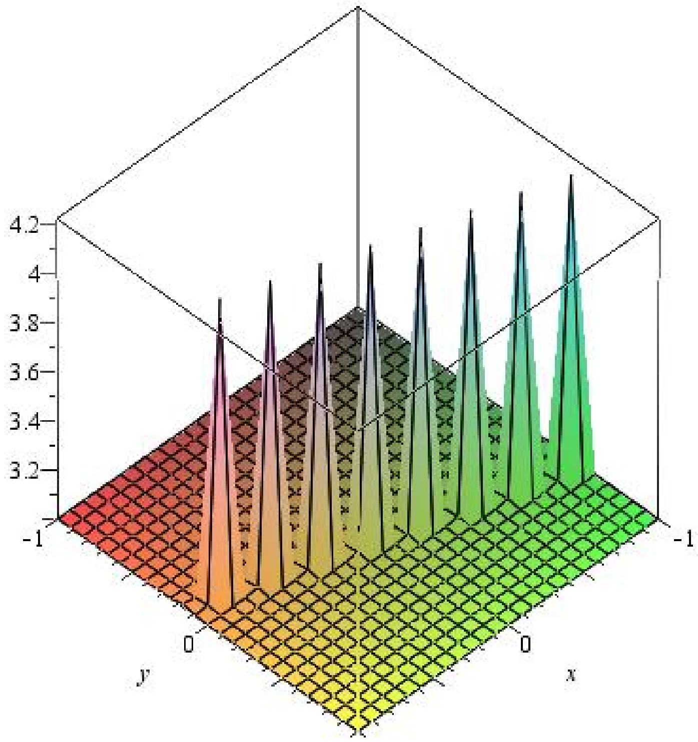

Figure 1: solution U_1 in eqno.(15) of eqno.(14) corresponding to $n_0 = 0, n_2 = 1, a_0 = 0, a_1 = 1, a_3 = 1, k = 3, v = 1$

where B_3 have the same meaning as before in eqno(3). Figure.1, Figure.2 represent the complexiton solution of (2+1) dimensional KdV equation. The shape is created by interacting hyperbolic with trigonometric function of solutions when t is constant.

Example 2 Consider the Hirota-Satsuma equation[32].

$$U_t + U_{xxx} + 6UU_x - 6VV_x = 0$$
$$V_t - 2V_{xxx} - 6UV_x = 0, \tag{23}$$

we handing eqno(23) as the same of eqno(12) with hypothesis eqno(13), $m = 2, n = 2$ according the homogeneous balance law.

$$U(\xi) = m_0 + m_1\varphi(\xi) + m_2\varphi^2(\xi)$$
$$V(\xi) = n_0 + n_1\varphi(\xi) + n_2\varphi^2(\xi), \tag{24}$$

where $m_0, m_1, m_2, n_0, n_1, n_2$ are arbitrary constants, they will be determined by the following work. Here $u(\xi)$ fulfills the ordinary sub-equation $\varphi'' = a + b\varphi + c\varphi^3$ $(\varphi(\xi) = \varphi(x - lt))$. With the help of solutions of $\varphi'' = a + b\varphi + c\varphi^3$, collecting the coefficients of all power of $\varphi(\xi)\varphi'(\xi)$ equate to zero after taking the eqno(24) into eqno(23).

Two series of free constants are obtained:

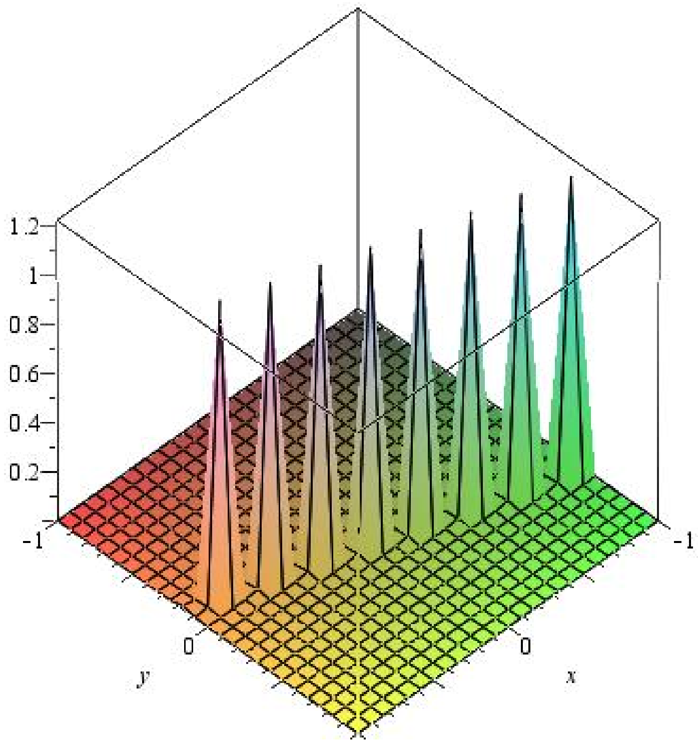

Figure 2: solution V_1 in eqno.(15) of eqno.(14) corresponding to $n_0 = 0, n_2 = 1, a_0 = 0, a_1 = 1, a_3 = 1, k = 3, v = 1$

Case 1: $v = v, m_0 = m_0, m_1 = 0, m_2 = RootOf(Z^2 - 2, label =_L 1)n_2$, $n_0 = -\frac{1}{4} RootOf(Z^2 - 2, label =_L 1)(l - 2m_0), n_1 = 0, n_2 = n_2$

Case 2:$v = \frac{-(n_1^2+2m_0m_2)}{m_2}, m_0 = m_0, m_1 = 0, m_2 = m_2, n_0 = n_0, n_1 = n_1, n_2 = 0$

Travelling exact solutions of eqno(23) in case 2:

$$U_1 = m_0 + m_2(a_0 + \frac{a_1(\tan(\xi) \pm \sec(\xi))(\sinh(B_3\xi) + \cosh(B_3\xi))}{a_3(\tan(\xi) \pm \sec(\xi)) + 1})^2$$
$$V_1 = n_0 + n_1(a_0 + \frac{a_1(\tan(\xi) \pm \sec(\xi))(\sinh(B_3\xi) + \cosh(B_3\xi))}{a_3(\tan(\xi) \pm \sec(\xi)) + 1}) \tag{25}$$

$$U_2 = m_0 + m_2(a_0 + \frac{a_1(\csc(\xi) - \cot(\xi))(\sinh(B_3\xi) + \cosh(B_3\xi))}{a_3(\csc(\xi) - \cot(\xi)) + 1})^2$$
$$V_2 = n_0 + n_1(a_0 + \frac{a_1(\csc(\xi) - \cot(\xi))(\sinh(B_3\xi) + \cosh(B_3\xi))}{a_3(\csc(\xi) - \cot(\xi)) + 1}) \tag{26}$$

$$U_3 = m_0 + m_2(a_0 + \frac{a_1(\frac{\tan(\xi)}{1\pm\sec(\xi)})(\sinh(B_3\xi) + \cosh(B_3\xi))}{a_3(\frac{\tan(\xi)}{1\pm\sec(\xi)}) + 1})^2$$
$$V_3 = n_0 + n_1(a_0 + \frac{a_1(\frac{\tan(\xi)}{1\pm\sec(\xi)})(\sinh(B_3\xi) + \cosh(B_3\xi))}{a_3(\frac{\tan(\xi)}{1\pm\sec(\xi)}) + 1}) \tag{27}$$

$$U_4 = m_0 + m_2(a_0 + \frac{a_1(\cot(\xi) \pm \csc(\xi))(\sinh(B_3\xi) + \cosh(B_3\xi))}{a_3(\cot(\xi) \pm \csc(\xi)) + 1})^2$$
$$V_4 = n_0 + n_1(a_0 + \frac{a_1(\cot(\xi) \pm \csc(\xi))(\sinh(B_3\xi) + \cosh(B_3\xi))}{a_3(\cot(\xi) \pm \csc(\xi)) + 1}) \tag{28}$$

$$U_5 = m_0 + m_2(a_0 + \frac{a_1(\sec(\xi) - \tan(\xi))(\sinh(B_3\xi) + \cosh(B_3\xi))}{a_3(\sec(\xi) - \tan(\xi)) + 1})^2$$
$$V_5 = n_0 + n_1(a_0 + \frac{a_1(\sec(\xi) - \tan(\xi))(\sinh(B_3\xi) + \cosh(B_3\xi))}{a_3(\sec(\xi) - \tan(\xi)) + 1}) \tag{29}$$

$$U_6 = m_0 + m_2(a_0 + \frac{a_1(\frac{\cot(\xi)}{1\pm\csc(\xi)})(\sinh(B_3\xi) + \cosh(B_3\xi))}{a_3(\frac{\cot(\xi)}{1\pm\csc(\xi)}) + 1})^2$$
$$V_6 = n_0 + n_1(a_0 + \frac{a_1(\frac{\cot(\xi)}{1\pm\csc(\xi)})(\sinh(B_3\xi) + \cosh(B_3\xi))}{a_3(\frac{\cot(\xi)}{1\pm\csc(\xi)}) + 1}) \tag{30}$$

$$U_7 = m_0 + m_2(a_0 + \frac{a_1(\frac{\tan(\xi)}{1-\tan^2(\xi)})(\sinh(B_3\xi) + \cosh(B_3\xi))}{a_3(\frac{\tan(\xi)}{1-\tan^2(\xi)}) + 1})^2$$
$$V_7 = n_0 + n_1(a_0 + \frac{a_1(\frac{\tan(\xi)}{1-\tan^2(\xi)})(\sinh(B_3\xi) + \cosh(B_3\xi))}{a_3(\frac{\tan(\xi)}{1-\tan^2(\xi)}) + 1}) \tag{31}$$

$$U_8 = m_0 + m_2(a_0 + \frac{a_1(\frac{\tan(\xi)}{1-\cot^2(\xi)})(\sinh(B_3\xi) + \cosh(B_3\xi))}{a_3(\frac{\tan(\xi)}{1-\cot^2(\xi)}) + 1})^2$$
$$V_8 = n_0 + n_1(a_0 + \frac{a_1(\frac{\tan(\xi)}{1-\cot^2(\xi)})(\sinh(B_3\xi) + \cosh(B_3\xi))}{a_3(\frac{\tan(\xi)}{1-\cot^2(\xi)}) + 1}), \tag{32}$$

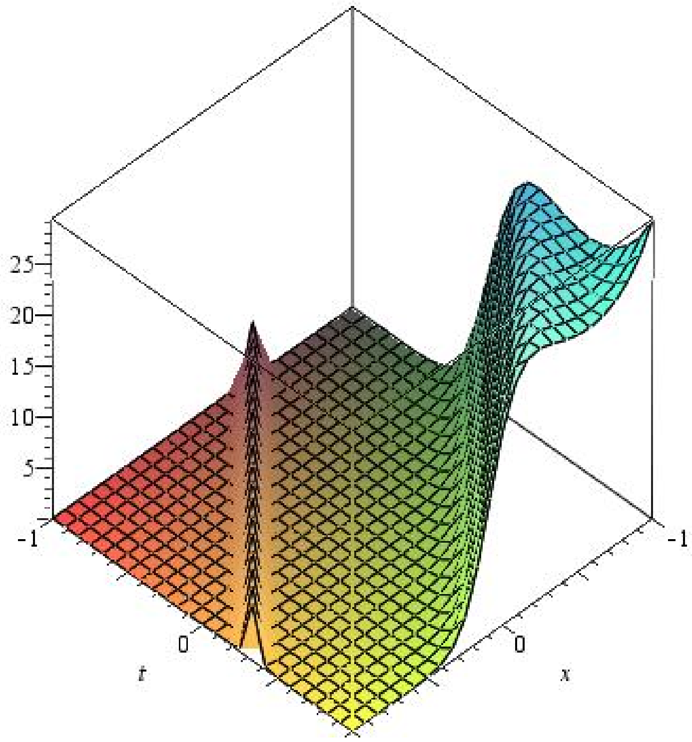

Figure 3: solution U_8 in eqno.(32) of eqno.(23) corresponding to $m_0 = 0, m_2 = 1, a_0 = 0, a_1 = 1, a_3 = 1, v = 1$

where $m_0, m_2, n_0, n_1, a_0, a_2, B_2, B_3,$ are arbitrary constants. Figure.3, Figure.4 represent the complexiton solution of Hirota-Satsuma equation. The shape is created by interacting hyperbolic with trigonometric function in solutions.

$U(\xi) = U(x - vt)$, v is propagation speed of soliton waves. Particular obtained solutions $U_i, V_i (1 \le i \le 8)$ work in concert with solutions of solvable ordinary eqno(1) which coefficients need to satisfying $a = -ca_0^3 - ba_0, b = b, c = c$. There some more interaction solutions of eqno(23), we ignore in this paper as in case 1 for simplification.

4 Conclusion and Discussion

In this paper, a novel auxiliary equation method is presented with a wide concrete applications. Two examples show that this method can be used to a large quantity of nonlinear evolution equations. It helps us to obtain interaction solutions of nonlinear evolution equations, which are not obtained in refs [31], [33]. This method will draw great attention due to the mixed function solutions of the novel auxiliary equation are obtained. This case is not appeared in previous methods such as the (G'/G)-expansion method. When we research these typical interaction solutions, complicate physical phenomena in nonlinear model systems will be study well.

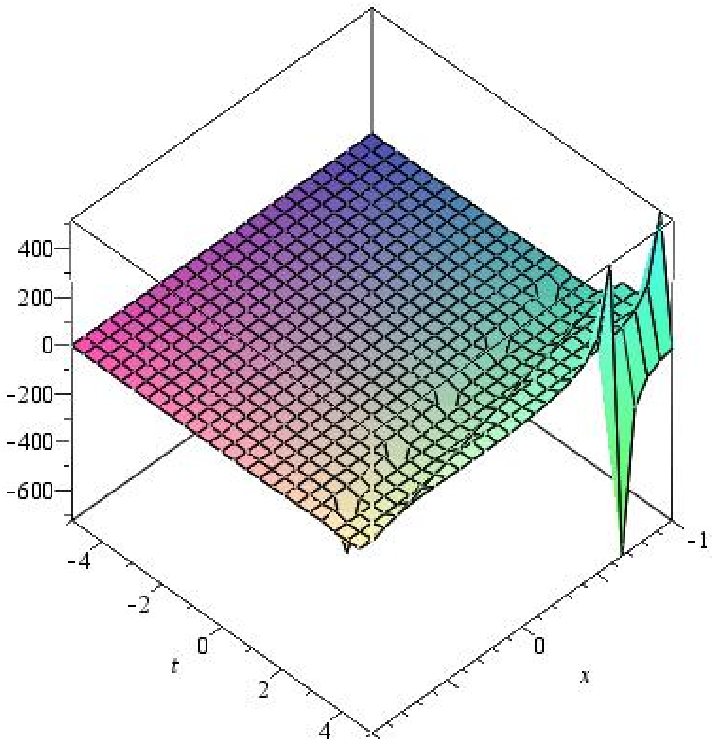

Figure 4: solution V_8 in eqno.(32) of eqno.(23) corresponding to $n_0 = 0, n_2 = 1, a_0 = 0, a_1 = 1, a_3 = 1, k = 3, v = 1$

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Competing Interests

The authors declare that no competing interests exist.

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