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Evaluating the Volatility Behaviour in Irish ISEQ Overall Index Using GARCH Models

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Authors' contributions

This work was carried out in collaboration between all authors. All authors together designed the study and wrote the protocol. Author AMA, as first author, managed the literature searches, analyses of the study and wrote the first draft of the manuscript. All authors read and approved the final manuscript.

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ABSTRACT

This paper aims to model the volatility of the Irish stock market (ISEQ) index price; data used in this article include the daily closing prices of ISEQ overall index, from January 1st, 2008 to March 28th, 2014. The data are observed to be naturally divided into three time frames; the first period from January 1st, 2008 to December 31st, 2009, the second from January 1st, 2010 to December 31st, 2011, and the third from January 1st, 2012 to March 28th, 2014. The volatility is modelled using symmetric and asymmetric Generalised Autoregressive Conditional Heteroscedasticity (GARCH) models including GARCH, EGARCH, TGARCH, PGARCH and FIEGARCH under the assumption that data follow a Normal distribution.
Comparisons, both of estimations and forecasts of the volatility between GARCH family models have been performed. In general, for the whole time frame (1/1/2008–28/3/2014) and for the first period, (as specified above), the FIGARCH (1,1) model performs better than the others but, for the second period, the PGARCH (1,1,1) model is preferred. For the third period, the best model is found to be EGARCH (1,1), so that the ISEQ overall index of volatility does appear to exhibit different behaviour in different periods.

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The empirical findings summary, for the GARCH forms that apply for the overall and three sub-periods, indicate that so-called *explosive volatility* is present in the ISEQ Overall index returns over the extended period.

Keywords: GARCH family models; volatility; volatility clustering and persistence.

1. INTRODUCTION

It is well known in financial markets that large changes tend to follow large changes, and small changes follow small; i.e. financial markets are sometimes *volatile* and sometimes *stable*. The sharply changing behaviour in financial markets is usually referred to as the “volatility” and is an important concept in different areas of financial theory and practice, such as risk management, portfolio selection, derivative pricing, etc. (Zivot and Wang [1]). In statistical terms, volatility is usually measured by the variance, (or standard deviation) of the series variable.

The theoretical framework that predicts stock market returns behaviour traditionally assumes that stock prices reflect all existing information on the market status, (asset pricing), (e.g. Fama [2,3]). Thus the predictability of stock returns relies on an equity efficient market, (with no information wasted or ignored) as well as underlying equilibrium, with constant expected returns. This is clearly unrealistic as, under the joint hypothesis of an efficient market and constant expected returns, predicting stock returns is not meaningful. Rather, the ability to make predictions depends on *variation*, in the stock returns over time, or from market inefficiency. Ability to predict stock returns does not imply the market is inefficient, but rather that the joint null hypothesis may involve an invalid asset-pricing model, (Klähn [4]). Typically, in empirical tests for expected returns, (influenced by market risk factors), different forecasting variables, both global and local, are employed as proxies for risk. Volatility has attracted considerable attention in the analysis of financial data because it is a numerical measure of the risk, faced by individual investors and financial institutions. It is well known that the volatility often fluctuates over time and tends to cluster in periods, i.e., high is followed usually by high, and low by low (Abdalla [5]). This phenomenon is denoted *volatility clustering*, and indicates that the volatility of the series is time-varying, (Lamoureux and Lastrapes [6]).

Modelling the volatility in financial time series is a major concern for economics, therefore, and many statisticians, economists and financial analysts have investigated the historical dependencies in the conditional variance of financial time series, with the aim of capturing and predicting the volatility. These include the Generalised Autoregressive Conditional Heteroscedasticity (GARCH) model and its extensions, which capture remarkable features of return series, such as *volatility clustering (*time varying conditional heteroscedasticity), *degree of persistence*, *volatility mean reversion* and high *kurtosis*, (Zivot and Wang [1]), Lim and Sek [7]). The degree of persistence measures consistent direction of movement, mean reversion implies that non-zero values are incorporated into the uncertainty and high kurtosis is well-known to be associated with sharp peaking at the *mean* and *heavy tails*. Unsurprisingly, the demand to forecast the absolute magnitude of returns (amongst other features) means that a considerable body of research now exists on volatility models.

Fayyad and Daly [8] investigated the volatility of market returns, dynamic conditional covariance and dynamic conditional correlation between the mature markets of the US and UK and the emerging markets of Kuwait and UAE, using a multivariate GARCH (MGARCH) model to identify the source and magnitude of volatility. Their findings showed that daily returns indicated volatility clustering as well as leverage effects, since both the regional market relation for Kuwait and UAE, as well as the global market relation of USA and UK, was enhanced during the financial crisis.

Subsequently, Ahmed and Suliman [9] used GARCH models of symmetric and asymmetric form to study the Khartoum Stock Exchange (KSE) (i.e. of The Sudan). Their results showed that the conditional variance process is highly persistent, and provides evidence on the existence of *risk premium,* which supports the hypothesis of positive correlation between volatility and the expected stock returns. The authors also demonstrated that asymmetric models, which take leverage into account, result in a better fit to the used data than symmetric ones. For the period 2005 to 2010, Arouri et al.

[10] investigated also the return linkages and volatility transmission between oil and stock markets in the Gulf Cooperation Council (GCC) countries. A recently generalised VAR-GARCH approach was used and results indicated the existence of substantial return and volatility spillovers, between world oil prices and GCC stock markets, of importance for international portfolio management in the presence of oil price risk.

Other recent work includes that of Goudarzi [11], who investigated mean reversion in the returns series for the Indian stock market, employing the Augmented Dickey- Fuller (ADF) test and a GARCH model to study market efficiency. The underlying series was found to be stationary (and therefore mean reverting) and the authors concluded that the Indian stock market is informationally weak-inefficient. Further, Bala and Asemota [12] studied the volatility in exchange–rate return series using GARCH models using monthly data for Nigerian Naira/US dollar, Nigerian Naira/British Pounds and Nigerian Naira/Euro. Estimates of GARCH model variants with ‘break’ in US dollar rates, (with break points exogenously determined), indicated volatility in the three currencies, but also a leverage effect, not highlighted in asymmetric models without volatility breaks. Both estimation and level of persistence were affected in models incorporating breaks, with the former improved and the latter reduced in most cases. In addition, the conditional volatility of the Saudi stock market returns were investigated using AR (1)-GARCH (1, 1) model, Kalyanaraman [13]. The author demonstrated that a symmetric GARCH (1, 1) model was adequate to estimate the volatility and found that the Saudi stock market returns are characterised by volatility clustering, exhibiting evidence of time-varying volatility, persistence and predictability. The Saudi stock market was also found to be highly reactive (nervous) to market fluctuations, an important factor in investor’s decisions relating to asset allocation and risk management strategies.

In what follows, the daily closing prices of the Irish stock market (ISEQ) index price are analysed over the period from January 1st, 2008 to March 28th, 2014. Breaks in the overall series are identified and GARCH type models (both symmetric and asymmetric) are used to explore volatility behaviour in the three data segments as well as in the overall data. No universal GARCH model was found to be applicable to all segmented sections of the data, but different model variants performed well, while explosive volatility does feature in the ISEQ overall index returns, with selected properties reported.

2. METHODOLOGY

A first autoregressive, conditionally heteroscedastic (ARCH) model, due to Engle [14], was subsequently generalised as the GARCH model, also proposed independently (Bollerslev [15], Taylor [16]). This has been shown to perform well for empirical financial time series and has given rise to variants, including asymmetric exponential GARCH, (Nelson [17]), asymmetric threshold GARCH, Zakoian[18]), asymmetric power GARCH, (Ding et al [19]), and asymmetric FIEGACH, (Bollerslev and Mikkelsen [20]). These univariate GARCH model types have been shown to be capable of modelling time-varying volatility and of capturing many stylised volatility features in financial time series.

Models are defined as follows:

2.1 Univariate GARCH

In the GARCH model, conditional variance is represented as a linear function of its own lags. The simplest model specification is the GARCH (1,1) model:

Mean equation $r_t = \mu + \varepsilon_t$ (1)

Variance equation $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$ (2)

where ω > 0, α ≥ 0, β ≥ 0, and,

r_t = returns of asset at time t.
μ = average returns.
$\varepsilon_t =$ residual returns, defined as: $\varepsilon_t = \sigma_t z_t$

where z_t are the standardised residual returns (i.e. values of an *i.i.d* random variable with zero mean and variance one), and σ_t^2 is the conditional variance. For GARCH (1,1), the constraints $\alpha_1 \geq 0$ and $\beta \geq 0$ are needed to ensure σ_t^2 is strictly positive.

In this model, the mean equation is written as a constant with an error term. The σ_t^2 term is a *conditional* variance, as it is the *one period ahead* forecast variance, (based on past information). The conditional variance equation (Equn. 2) consists of:

- A constant term: α_0
- ε_{t-1}^2 (the ARCH term). Information on volatility from the previous period, measured as the lag of the squared residual from the mean equation (Equn. 1).
- σ_{t-1}^2 (the GARCH term). The forecast variance from the previous period.

This equation models the time-varying nature of the volatility of the residuals generated from the mean equation (Equn. 1). This specification is often interpreted in a financial context, in terms of an agent (or trader) predicting the current period variance by forming a weighted average comprised of a long term average (the constant), the forecast variance from the previous period (the GARCH term), and information about the volatility observed in the previous period (the ARCH term). If the asset return is unexpectedly large in either the upward or the downward direction, then the trader increases the estimate of the variance for the next period, (Ahmed and Suliman [9]). The EGARCH model variant follows from this.

2.2 The Exponential GARCH (EGARCH) Model

The exponential nature of the conditional variance in the EGARCH model captures the effect of external unexpected shocks on the predicted volatility. The EGARCH (1, 1) model is formulated as:

$$\ln \sigma_t^2 = \alpha_0 + \beta_1 \ln \sigma_{t-1}^2 + \alpha_1 \left\{ \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right\} - \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \quad (3)$$

where γ is the asymmetric response or leverage parameter. The sign of γ is expected to be positive in most empirical cases, so that a negative shock increases future volatility or uncertainty while a positive shock decreases the effect on future uncertainty.

2.3 The Threshold GARCH (TGARCH) Model

The threshold-GARCH process allows the effect of good and bad news (negative and positive return shocks) on the volatility to be analysed, i.e. can be used to incorporate *leverage* effects. TGARCH (1, 1) is the standard GARCH (1,1) model with the addition of an asymmetric threshold term:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma d_{t-1} \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (4)$$

where d_{t-1} is a dummy variable, that is:

$$d_{t-1} = \begin{cases} 1 \ \ if \ \varepsilon_{t-1} < 0, & bad\ news \\ 0 \ \ if \ \varepsilon_{t-1} \geq 0, & good\ news \end{cases}$$

The coefficient γ is known as the asymmetric or leverage term such that, when $\gamma = 0$, the model collapses to the standard GARCH model (Equn. 2). For positive shock (i.e. good news) the effect on volatility is α_1, while for bad news, the effect on volatility is $\alpha_1 + \gamma$. Hence, if γ is significant and positive, then negative shocks have a larger effect on σ_t^2 than positive shocks, (as for EGARCH).

2.4 The Power GARCH (PGARCH) Model

In the power GARCH model, the standard deviation is used instead of the variance and an optional parameter γ can be added to account for asymmetry in modelling. The asymmetric PGARCH (1,1,δ) model specifies σ_t to be of the following form:

$$\sigma_t^\delta = \alpha_0 + \beta_1 \sigma_{t-1}^\delta + \alpha_1 (|\varepsilon_{t-1}| - \gamma_1 \varepsilon_{t-1})^\delta \quad (5)$$

where α_1 and β_1 are the standard ARCH and GARCH parameters, γ_1 and δ are the leverage parameter and parameter for the power term respectively, with $\delta \in (0,2]$, and $|\gamma_1| \leq 1$. The symmetric model sets $\gamma_1 = 0$, and when δ = 2, the above equation (Equn. 5) becomes a classic GARCH (1,1) and allows for leverage effects while, when δ = 1, the conditional standard deviation is estimated instead.

2.5 The Fractionally Integrated Exponential GARCH (FIEGARCH) Model

The basic GARCH (*p*,*q*) model can be written also as an ARMA(*p*,*q*) model in terms of squared residuals:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \quad (6)$$

which can be rewritten directly as:

$$\emptyset(L) \varepsilon_t^2 = \alpha_0 + b(L) u_t \quad (7)$$

Where

$$u_t = \varepsilon_t^2 - \sigma_t^2$$
$$\emptyset(L) = 1 - \emptyset_1 L - \emptyset_2 L^2 - \cdots - \emptyset_m L^m$$
$$b(L) = 1 - b_1 L - b_2 L^2 - \cdots - b_q L^q$$

with m= max(p, q) and $\emptyset_i = \alpha_i + \beta_i$. The *high persistence* in GARCH models suggests that the polynomial $\emptyset(z) = 0$ may have a unit root, in which case the model becomes the *integrated GARCH (IGARCH)* model (Zivot and Wang [1]). To allow for *high persistence* and *long memory* in the conditional variance and to avoid the complications of IGARCH models, it is possible to extend the ARMA(m, q) to a fractional autoregressive moving average, FARIMA(m, d, q) process, as follows:

$$\emptyset(L)(1-L)^d\varepsilon_t^2 = \alpha_0 + b(L)u_t \quad (8)$$

where all the roots of $\emptyset(z) = 0$ and b(z) = 0 lie outside the unit circle. When d = 0, this reduces to the usual GARCH model; when d=1, this becomes the IGARCH model; when 0<d<1, the fractionally differenced squared residuals, $(1-L)^d\sigma_t^2$, follow a stationary ARMA(m, q) process. The above FARIMA process for ε_t^2 can be rewritten then in terms of the conditional variance σ_t^2:

$$b(L)\sigma_t^2 = \alpha_0 + [b(L) - \emptyset(L)(1-L)^d]\varepsilon_t^2 \quad (9)$$

Baillie et al. [21] refer to the above model as the *fractionally integrated* GARCH, or FIGARCH (m, d, q) model. Furthermore, when 0 <d < 1, the coefficients in $\emptyset(L)$ and b(L) capture the short run dynamics of volatility, while the fractional difference parameter 'd' models the long run characteristics of volatility.

The FIGARCH model directly extends the ARMA representation of squared residuals, which obtains for the GARCH model, to a fractionally integrated model. However, to guarantee that a general FIGARCH model is stationary and the conditional variance σ_t^2 is always positive, complicated and intractable restrictions usually have to be imposed on the model coefficients (see Zivot and Wang [1]).

Noting that an EGARCH model can be represented as an ARMA process in terms of the logarithm of conditional variance and thus always guarantees that the conditional variance is positive, Bollerslev and Mikkelsen [20] proposed the following fractionally integrated EGARCH (FIEGARCH) model: FIEGARCH(1,1,d)

$$\emptyset(L)(1-L)^d ln\sigma_t^2 = \alpha_0 + \beta_1|z_{t-1}| + \gamma z_{t-1} \quad (10)$$

where $\emptyset(L)$ is defined as earlier for the FIGARCH model, $\gamma \neq 0$ allows the existence of leverage effects, and z_t is the standardized residual:

$$z_t = \frac{\varepsilon_t}{\sigma_t} \quad (11)$$

The authors also showed that the FIEGARCH model is stationary if 0 < d < 1.

2.6 Choosing a Model

Choosing the appropriate model is not always straightforward, of course, and often requires further information, based on the likelihood of a particular model form producing the empirical results. The *Akaike Information Criterion* (AIC) and *Bayesian Information Criterion* (BIC) provide measures of the quality of the models chosen, are both based, in part, on the likelihood function and are closely related to each other. For large n, the AIC and the BIC are calculated as follows:

$$AIC = 2.k - 2.\ln L \quad (12)$$

$$BIC = -2.\ln L + k.\ln(n) \quad (13)$$

where,

- n = the number of observations or equivalently, the sample size;
- k= the number of free parameters to be estimated. If the model under consideration is a linear regression, k is the number of regressors, including the intercept;
- L= the maximized value of the likelihood function of the model.

3. DATA DESCRIPTION AND EMPIRICAL RESULTS

3.1 Data Description

The daily closing prices of the ISEQ overall index, from January 1st, 2008 to March 28th, 2014, made available by the Irish Stock Exchange (www.ise.ie), are used. In this study, daily returns (r_t) refer to continuously compounded returns given by the first difference in the natural logarithm of closing prices of the ISEQ overall index on successive days:

$$r_t = log\left(\frac{P_t}{P_{t-1}}\right) \quad (14)$$

where P_t and P_{t-1} are the closing prices at day t and t-1, respectively.

Compounding returns are also assumed to be normally distributed, (time additive property), whereas simple or discrete returns are portfolio-, but not time-, additive. The stationarity in the mean of the series is assessed by the unit root test (Table 1). The non-stationarity in the variance of returns is accounted for by the conditional variance in the GARCH model family and the stationarity condition for these models. The main value of the so-called log-return, which is the log of the price ratio, is that it is essentially a proxy for the percentage change in the price, so that the average log-return over a period of time is approximately equal to the average percentage change in price, if the stock price does not change by much. However, when the price is very volatile, the average log return can be very different to the average percentage change and far more detailed investigation of the volatility is needed. In consequence, we report in detail on the results for the GARCH models applied to the r_t, as defined (Equn. 14).

From Table 1, which provides a statistical description of the returns and squared returns in different periods, it can be seen that the return series are negatively skewed and, with the exception of the returns for the third period (Jan, 1st 2012-March 28th 2014), have *fat tails* compared to the Normal distribution. Tests indicate that the returns series are, in fact, *not Normally distributed* for returns of the first periods or over the whole time frame. The unit root test shows, however, that all the series are stationary.

From Table 1, it can be seen that the means of all series are positive, except for that of the returns in the whole period. According to the sample standard deviations, the returns series in the third period is the least volatile, with a standard deviation of 0.00939, while the returns series in the first period is the most volatile, with a standard deviation of 0.02608. The standard deviations for the returns in the whole and second period are 0.01797 and 0.01509, respectively, suggesting that the volatility of these two series is almost the same. This table also demonstrates that returns series for all four cases do not follow the Normal distribution and autocorrelation is zero. A plot of the daily prices against time illustrates these points, Fig. 1.

The figure shows daily prices of the original data used in this study for the time period January 1st, 2008 to March 28th, 2014 and it is evident that there is no general trend. However, three different sections can be identified in the plot. A clear and strong decrease in prices occurs over the period from January 1st, 2008 to December 31st, 2009, prices are relatively stable from January 1st, 2010 to December 31st, 2011, while from January 1st, 2012 to March 28th, 2014, a gradual increase is observed. The three identified sections or segments of the data were modelled separately, in addition to the analysis for the complete series. The returns and squared returns series, plotted against time, provide an illustration of the level of volatility over the entire time period, Fig. 2. It is clear that the first period is largely responsible for the most vigorous change.

Table 1. Summary statistics for returns and squared return series for the four different periods

Measures	Time series							
	The whole period		The 1st period		The 2nd period		The 3ed period	
	Return	Sq. Ret	Return	Sq. Ret	Return	Sq. Ret	Return	Sq. Ret
Minimum	-0.13960	0.0000	-0.13960	0.0000	-0.05950	0.0000	-0.0295	0.0000
Maximum	0.09730	1.949E-2	-2.105E-4	3.29E-4	-7.46E-5	2.27E-4	9.51E-4	8.89E-5
Mean	-0.00021	3.229E-4	0.0973	1.949E-2	0.0757	0.00573	0.02830	8.70E-4
St. Dev	0.01797	9.158E-4	0.02608	0.00148	0.01509	4.60E-4	9.39E-3	1.35E-4
Skewness	-0.48076	9.7330	-0.32093	6.34677	-0.10216	5.51541	-0.1650	2.58792
Kurtosis	6.04051	1.498E2	2.68571	60.63300	2.13689	47.64080	0.38495	8.01553
Autocorrelation Test	NO	YES	NO	YES	NO	YES	NO	YES
Normality Test	NO	NO	NO	NO	YES	NO	YES	NO
Unit Root Test	NO	NO	NO	NO	NO	NO	NO	NO
ARCH Effect	YES	YES	YES	YES	YES	NO	YES	NO
Long memory	NO	YES	NO	YES	NO	NO	NO	NO

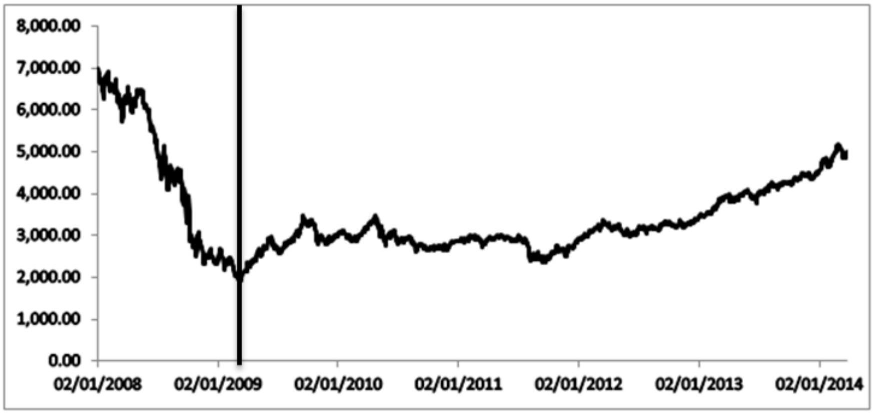

Fig. 1. The distribution of the prices of ISEQ overall index, with first series break shown

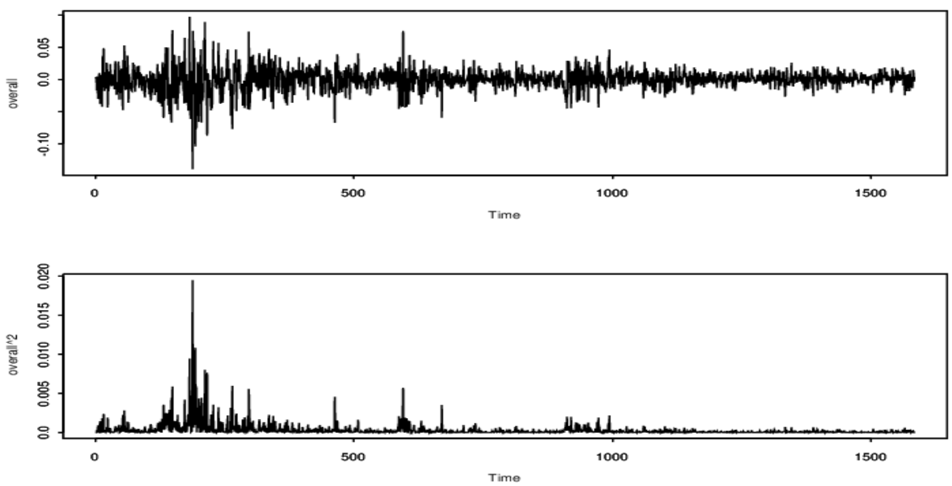

Fig. 2. The distribution of the returns and squared returns of ISEQ overall index

Table 2. Estimated parameters of all Time-varying models

Estimated coefficients	Conditional variance equation				
	GARCH(1,1)	**EGARCH(1,1)**	**TGARCH(1,1)**	**PGARCH(1,1,1)**	**FIEGARCH(1,1)**
M	6.80E-4*	0.000254	3.870E-4	0.000338	3.801E-4*
α_0	1.70E-6*	-0.293351*	2.231E-6*	0.000187*	-0.141512*
ARCH (α_1)	0.07940*	0.173752*	0.039640*	0.086682*	0.145083*
GARCH (β_1)	0.91500*	0.981470*	0.913600*	0.919412*	0.631637*
$\alpha_1 + \beta_1$	0.99440	1.155222	0.953240	1.006094	0.776720
Leverage	-------	-0.406586*	-------	-0.384317*	-0.059316*
GAMMA	-------	-------	7.479E-2*	-------	--------
Fraction	-------	-------	-------	-------	0.592094*

** indicates the parameter is statistically significant at 5%.*

- *Mean Equation is overall~1 for all models.*
- *Conditional distribution is Gaussian (Normal) for all models*

3.1.1 The complete series

The results of estimation, selection and comparison, as well as diagnostics for the returns series over the whole period are given in this subsection:

Table 2 shows that *only* the parameters of GARCH (1,1) and FIEGARCH (1,1) models are significant and also shows (highlighted row) that $\boldsymbol{\alpha_1 + \beta_1}$ in these two models is less than one, meaning that the stationary condition holds.

Table 3 shows that the values of the selection criteria, (AIC and BIC), did not vary much amongst the tested models, so the most parsimonious, namely FIEGARCH (1,1) is chosen. The analysis of residuals from this model indicates that there is neither autocorrelation nor ARCH effect in the residuals and the squared residuals, and also shows that the residuals are *Normally distributed*.

Further, the bootstrap confidence intervals plot, Fig. 3, illustrates that the FIEGARCH (1,1) model is a good fit to the whole time frame data.Corresponding results of estimation, selection and comparison, as well as diagnostics for the returns series in each period are given in the next three subsections.

3.1.2 The first period

Table 4 shows that only the parameters of the FIEGARCH (1,1) model are significant and also shows that with ($\boldsymbol{\alpha_1 + \beta_1}$) < 1, the stationary condition also holds for this model.

Table 5 shows that the values of the selection criteria (AIC and BIC) did not vary much for the tested models, so the most parsimonious, namely, FIEGARCH (1,1) is chosen. Again, analysis of residuals from this model indicates no autocorrelation or ARCH affects in either the residuals or the squared residuals and the Normal distribution applies. In general, a good fit is again obtained with this model for the first period, Fig. 4.

Table 3. AIC, BIC, autocorrelation, normal and ARCH effect tests for competing models

Measures	**Conditional variance equation**				
	GARCH(1,1)	**EGARCH(1,1)**	**TGARCH(1,1)**	**PGARCH(1,1,1)**	**FIEGARCH(1,1)**
AIC	- 8943.323	- 8956.847	-8957.662	-8960.795	- 8967.021
BIC	- 8921.852	- 8930.009	-8930.823	-8933.957	-8934.814
Autocorrelation in Residuals	NO	NO	NO	NO	NO
Autocorrelation in Sq. Residuals	NO	NO	NO	NO	NO
Normality test for Residuals	Normal	Normal	Normal	Normal	Normal
ARCH effect in Residuals	NO	NO	NO	NO	NO
ARCH effect in Sq. Residuals	NO	NO	NO	NO	NO

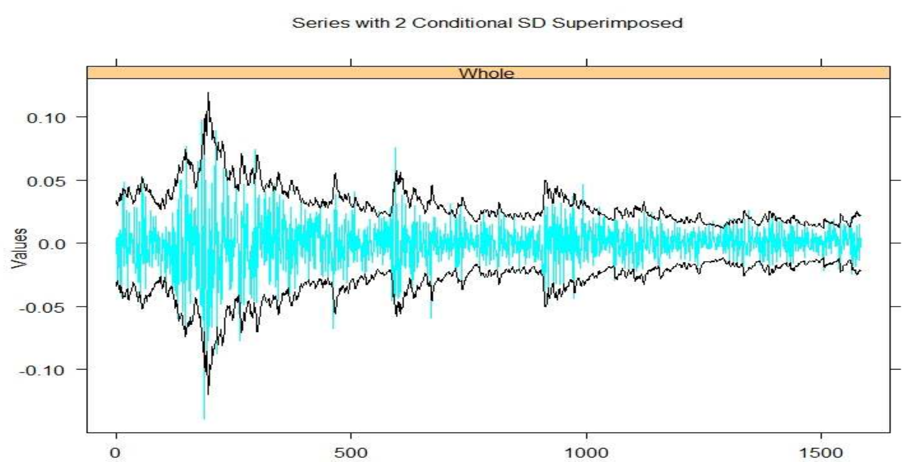

Fig. 3. 95% Bootstrap confidence intervals for residuals from FIEGARCH (1,1) for the whole time period

Table 4. The estimated parameters of all Time-varying models

Estimated coefficients	Conditional variance equation				
	GARCH(1,1)	EGARCH(1,1)	TGARCH(1,1)	PGARCH(1,1,1)	FIEGARCH(1,1)
M	-0.000409	-0.001108	-0.0008505	-0.0010831	-0.001092*
α_0	1.137E-5*	-0.361088*	1.122E-5*	0.0004302*	-0.093576*
ARCH (α_1)	0.089373*	0.150332*	0.0422196	0.0694307*	0.089675*
GARCH (β_1)	0.893777*	0.967835*	0.9003486*	0.9270629*	0.781910*
$\alpha_1 + \beta_1$	**0.983150**	**1.118167**	**0,9425682**	**0.9964936**	**0.871585**
Leverage	-------	-0.568536*	--------	-0.5863963*	-0.053453*
Gamma	-------	-------	0.0747593*	-------	--------
Fraction	-------	-------	-------	-------	0.582207*

** indicates the parameter is statistically significant at 5%.*

- *Mean Equation is period1~1 for all models.*
- *Conditional distribution is Gaussian for all models*

Table 5. AIC and BIC, autocorrelation, normal and ARCH effect tests for competing models

Measures	Conditional variance equation				
	GARCH(1,1)	EGARCH(1,1)	TGARCH(1,1)	PGARCH(1,1,1)	FIEGARCH(1,1)
AIC	-2356.611	-2361.308	-2359.445	-2363.538	-2365.483
BIC	-2339.697	-2340.165	-2328.302	-2342.395	-2345.112
Autocorrelation in Residuals	NO	NO	NO	NO	NO
Autocorrelation in Sq. Residuals	NO	NO	NO	NO	NO
Normality test for Residuals	Normal	Normal	Normal	Normal	Normal
ARCH effect in Residuals	NO	NO	NO	NO	NO
ARCH effect in Sq. Residuals	NO	NO	NO	NO	NO

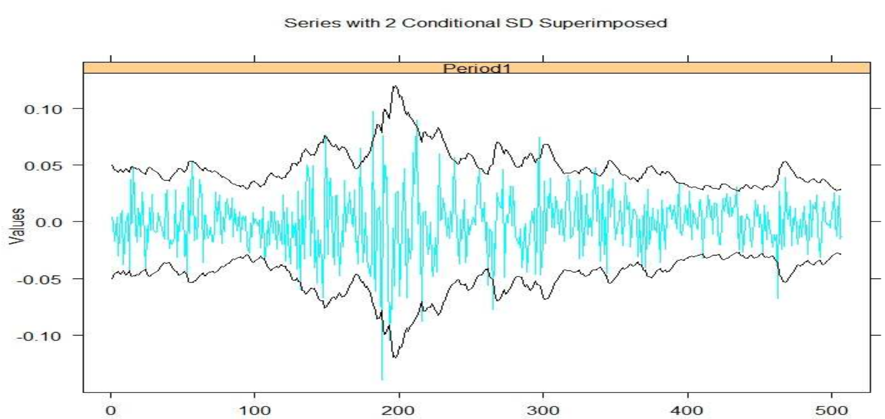

Fig. 4. 95% bootstrap confidence intervals for residuals from FIEGARCH (1,1) for the first period

3.1.3 The second period

Table 6 shows that only parameters of the PGARCH (1,1) model are significant and, given ($\alpha_1 + \beta_1$) < 1 , that the stationary condition also holds for this model.

3.1.4 The third period

Table 8 shows that only parameters of EGARCH (1,1) and PGARCH (1,1,1) models are all significant, with the stationary condition satisfied for EGARCH (1,1) only. Again, little variation is seen in the selection criteria (AIC and BIC values, Table 9) for the tested models, with EGARCH (1,1) chosen as the most parsimonious. The analysis of residuals from this model once again indicates that all criteria, as before, are satisfied. The EGARCH (1,1) model also provides a good fit to the data in the third period, Fig. 6.

Table 6. The estimated parameters of all Time-varying models

Estimated Coefficients	Conditional variance equation				
	GARCH(1,1)	**EGARCH(1,1)**	**TGARCH(1,1)**	**PGARCH(1,1,1)**	**FIEGARCH(1,1)**
M	4.507E-4	-2.332E-4	3.771E-5	-1.423E-4*	-1.195E-4
α_0	1.501E-5*	-0.888939*	1.482E-5*	9.351E-4*	-0.297368*
ARCH (α_1)	0.163055*	0.2580919*	0.064569*	0.1392135*	0.2106545*
GARCH (β_1)	0.772826*	0.9197883*	0.781030*	0.8248930*	0.7767394*
$\alpha_1 + \beta_1$	**0.935881***	**1.1778802***	**0.845599***	**0.9641065***	**0.9873939***
Leverage	------------	-0.474561	------------	-0.4967299*	-0.091661*
GAMMA	------------	-----------	0.178461*	-------------	-------------
Fraction	------------	----------	----------	-------------	0.373402

** indicates the parameter is statistically significant at 5%.*

- *Mean Equation is overall~1 for all models.*
- *Conditional distribution is Gaussian for all models*

Table 7. AIC, BIC, autocorrelation, normal and ARCH effect tests for competing models

Measures	Conditional variance equation				
	GARCH(1,1)	**EGARCH(1,1)**	**TGARCH(1,1)**	**PGARCH(1,1,1)**	**FIEGARCH(1,1)**
AIC	-2892.841	-2900.352	-2899.077	-2902.288	-2900.265
BIC	-2875.919	-2879.199	-2877.925	-2881.135	-2874.882
Autocorrelation in Residuals	NO	NO	NO	NO	NO
Autocorrelation in Squared Residuals	NO	NO	NO	NO	NO
Normality test for Residuals	Normal	Normal	Normal	Normal	Normal
ARCH effect in Residuals	NO	NO	NO	NO	NO
ARCH effect in Squared Residuals	NO	NO	NO	NO	NO

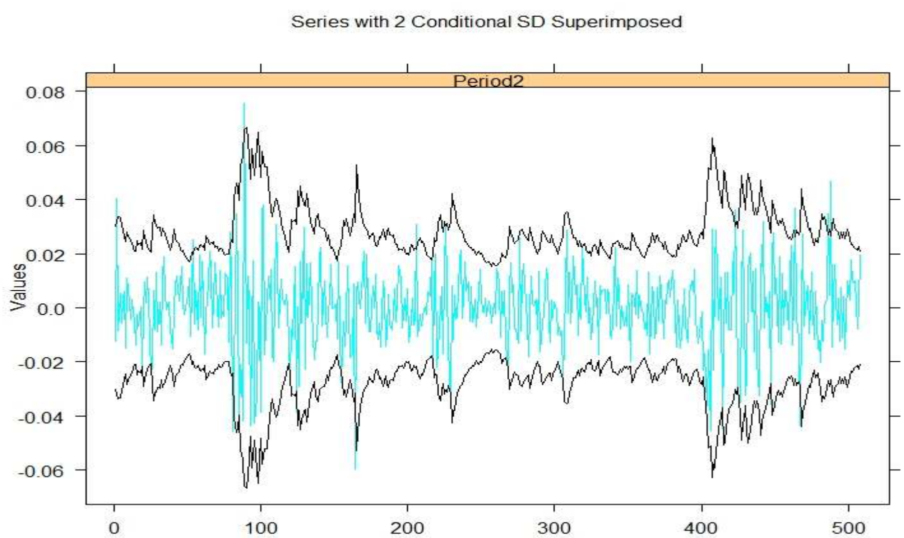

Fig. 5. 95% bootstrap confidence intervals for residuals from PGARCH (1,1,1) for the second period

Table 8. The estimated parameters of all Time-varying models

Estimated Coefficients	Conditional variance equation			
	GARCH(1,1)	**EGARCH(1,1)**	**TGARCH(1,1)**	**PGARCH(1,1,1)**
M	9.999E-4*	7.923E-4*	8.705E-4*	7.889E-4*
α_0	1.712E-6	-2.148038*	8.161E-6*	8.098E-4*
ARCH (α_1)	3.186E-2*	0.114358*	-2.026E-2	0.482972*
GARCH (β_1)	9.491E-1*	0.780836*	8.661E-1*	0.8754322*
$\alpha_1 + \beta_1$	**0.97996**	**0.89519**	**0.84580**	**1.35850**
Leverage	-------	-0.948110*	--------	-0.713375
Gamma	-------	-------	1.153E-1*	-------

** indicates the parameter is statistically significant at 5%.*

- *Mean Equation is period3~1 for all models.*
- *Conditional distribution is Gaussian for all models*

Table 9. AIC and BIC, autocorrelation, normal and ARCH effect tests for competing models

Measures	Conditional variance equation			
	GARCH(1,1)	**EGARCH(1,1)**	**TGARCH(1,1)**	**PGARCH(1,1,1)**
AIC	-3711.062	-3716.847	-3714.116	-3707.717
BIC	-3693.68	-3695.118	-3692.388	-3685.989
Autocorrelation in Residuals	NO	NO	NO	NO
Autocorrelation in Sq. Residuals	NO	NO	NO	NO
Normality test for Residuals	Normal	Normal	Normal	Normal
ARCH effect in Residuals	NO	NO	NO	NO
ARCH effect in Sq. Residuals	NO	NO	NO	NO

Table 10. Comparisons between the fitted models for the four different periods

Measures	Total period	First period	Second period	Third period
	FIEGARCH(1,1)	**FIEGARCH(1,1)**	**PGARCH (1,1,1)**	**EGARCH(1,1)**
$\alpha_1 + \beta_1$	0.776720	0.871585	0.964106	0.895194
Daily variance with	3.229E-04	6.802E-04	2.277E-04	8.817E-05
Annualized volatility rate of	0.2852 %	0.4140 %	0.2395 %	0.1491 %
Volatility half-life[1]	4 days	6 days	20 days	7 days

- *The process is mean reverting if ($\alpha_1 + \beta_1$) <1*
- *The process is covariance stationary if 0 < ($\alpha_1 + \beta_1$) <1, with degree of persistence in the conditional variance high as the sum is close to one.*
- *The volatility half-life is computed as $\frac{log[(\alpha_1+\beta_1)/2]}{log(\alpha_1+\beta_1)}$.*

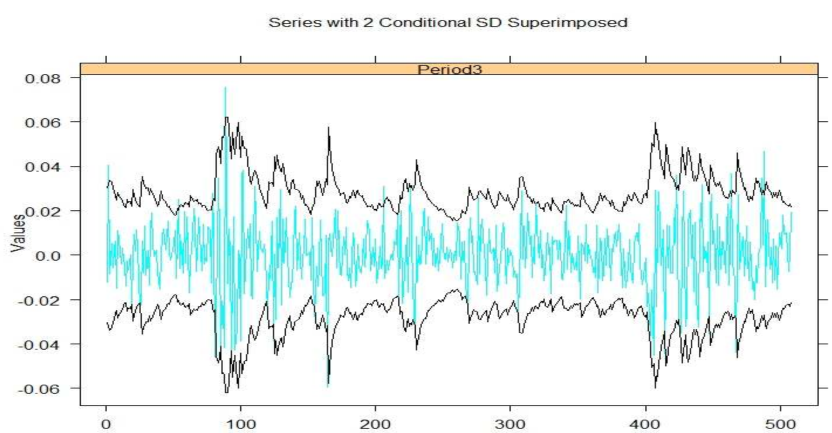

Fig. 6. 95% bootstrap confidence intervals for residuals from EGARCH (1,1) for the third period

[1] *Half-life measures how many days pass until half of the initial shock is absorbed by the variance.*

3.1.5 General comparison

Comparisons between the fitted models for the four different periods, including overall or total, are made on the basis of mean reversion, degree of persistence and the volatility half-life and findings are illustrated in Table 10.

From the above table, it can be seen that the sum of the ARCH and GARCH coefficients ($\alpha_1 + \beta_1$) for the total, first, second and third periods are 0.776720, 0.871585, 0.964106 and 0.895194, respectively (all < 1), which supports the assumption of covariance stationarity as well as volatility persistence in the data). It also indicates that the half-life of a volatility shock (or volatility shock duration) for the overall, first, second and third periods are, approximately, 4, 6, 20 and 7 days.

4. CONCLUSIONS

In summary, this study attempts to capture the picture of volatility in the Irish stock market, using symmetric and asymmetric GARCH models, (GARCH, EGARCH, PGARCH, TGARCH, PGARCH and FIEGARCH). These were applied to the complete data set (1/1/2008-28/3/2014) and to three distinctly identified sub-periods, i.e. (1/1/2008-31/12/2009), (1/1/2010-31/12/2011) and (1/1/2012-28/3/2014). Results show that performances of individual model types vary across the different time periods. In general, the FIEGARCH (1,1) model performed best for the total and first periods while the PGARCH(1,1,1) model worked well over the second period and the EGARCH(1,1) model best captured the volatility in the third.

Our findings also indicate that there are different patterns between the returns in the different periods, with no universal model applicable. Finally, the results show that the volatility in returns for each period exhibits different *degree of persistence*, *volatility clustering* and *mean reversion*. The overall data period corresponds, of course, to major perturbations in the financial markets, but it is interesting to note that extreme instability was relatively short-lived, although recovery was not immediate. The relative model fits suggest that GARCH model variants interpret, and even anticipate, this staged behaviour quite well.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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