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Modified Ratio-Cum-Product Estimators of Population Mean Using Two Auxiliary Variables

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Authors' contributions

This work was carried out in collaboration among all authors. Author JOM designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors ENA, YAE, MAY and AA managed the analyses of the study. Author MAH managed the literature searches. All authors read and approved the final manuscript.

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ABSTRACT

A percentile is one of the measures of location used by statisticians showing the value below which a given percentage of observations in a group of observations fall. A family of ratio-cum-product estimators for estimating the finite population mean of the study variable when the finite population mean of two auxiliary variables are known in simple random sampling without replacement (SRSWOR) have been proposed. The main purpose of this study is to develop new ratio-cum-product estimators in order to improve the precision of estimation of population mean in sample random sampling without replacement using information of percentiles with two auxiliary variables. The expressions of the bias and mean square error (MSE) of the proposed estimators were derived by Taylor series method up to first degree of approximation. The efficiency conditions under which the proposed ratio-cum-product estimators are better than sample man, ratio estimator, product estimator and other estimators considered in this study have been established. The numerical and empirical results show that the proposed estimators are more efficient than the sample mean, ratio estimator, product estimator and other existing estimators.

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Keywords: Percentiles; ratio estimator; product estimator; population mean; study variable; kurtosis.

1. INTRODUCTION

A percentile is one of the measures of location used by statisticians showing the value below which a given percentage of observations in a group of observations fall. Percentiles play an important part in descriptive statistics and their use is well recommended. Percentiles divide a set of ordered data into hundredths. Median (M_d) is the 50 th percentile. In a situation where auxiliary information is available, it is possible to devise suitable ways of using it in obtaining the sample strategies which are better than those in which no such information is used. When the information on an auxiliary variable X is known, a ratio, product or linear regression estimator could be employed for the estimation of finite population mean or variance. Cochran [1] made an important contribution to the modern sampling theory by suggesting methods of using the auxiliary information for the purpose of estimation of population mean so as to increase the precision of the estimates. Cochran [1] developed the ratio estimator to estimate population mean or the total of the study variable.

Many authors have developed ratio and product type estimators for estimating population mean of study variable like Upadhayaya and Singh [2], Abu-Dayyeh [3], Singh et al. [4], Kadilar and Cingi [5], Tailor et al. [6], Jeelani et al. [7], Gupta and Yadav [8], Muili et al. [9], Muili and Audu [10], Muili et al. [11], etc. None of the above authors have used percentiles as population parameters for estimating population mean of study variable.

The purpose of this study is to develop new ratio-cum-product estimators to improve the precision of estimation of population mean in sample random sampling without replacement using information of percentiles with two auxiliary variables.

Consider $U=\{U_1,U_2,U_3,...,U_N\}$ be a finite population having N units and each $U_i=(X_i,\,Y_i),\ i=1,2,3,...,N$ has a pair of values. Y is the study variable and X_1 and X_2 are the two auxiliary variables which are correlated with Y . Let $y=\{y_1,y_2,...,y_n\}$, $x_1=\{x_{11},x_{12},...,x_{1n}\}$, and $x_2=\{x_{21},x_{22},...,x_{2n}\}$ be n sample values. $\bar{y}$, $\bar{x}_1$ and $\bar{x}_2$ are the sample means of the study and auxiliary variables respectively. S_y^2 and S_{xi}^2 be the mean square population of Y and X_i respectively and s_y^2 and s_{xi}^2 be respective sample mean square based on the random sample of size n drawn without replacement. N : Population size, n: Sample size, $\bar{Y},\bar{X}_i$: Population means of study and auxiliary variables ρ_{yx_i} : Coefficient of correlation, C_y,C_{xi} : Coefficient of variations of study and auxiliary variables, $\beta_{2(x_i)}$: Coefficient of Kurtosis of auxiliary variables, $M_d(x_i)$: Median of the auxiliary variables. Sampling fraction(f) is the ratio of sample size to population size. Percentage Relative Efficiency (PRE) is a statistical tool used to measure and ascertain the efficiency of one estimator over another.

$$\bar{X}_1=\frac{1}{N}\sum_{i=1}^{N}X_{1i},\ \bar{X}_2=\frac{1}{N}\sum_{i=1}^{N}X_{2i},\ \bar{Y}=\frac{1}{N}\sum_{i=1}^{N}Y_i,\ \bar{x}_1=\frac{1}{n}\sum_{i=1}^{n}x_{1i},\ \bar{x}_2=\frac{1}{n}\sum_{i=1}^{n}x_{2i},\ \bar{y}=\frac{1}{n}\sum_{i=1}^{n}y_i,$$

$$s_y^2=\frac{1}{n-1}\sum_{i=1}^{n}\left(y_i-\bar{y}\right)^2,\ \ s_{x1}^2=\frac{1}{n-1}\sum_{i=1}^{n}\left(x_{1i}-\bar{x}_1\right)^2,\ s_{x2}^2=\frac{1}{n-1}\sum_{i=1}^{n}\left(x_{2i}-\bar{x}_2\right)^2,\ S_y^2=\frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i-\bar{Y}\right)^2,$$

$$S_{x1}^2=\frac{1}{N-1}\sum_{i=1}^{N}\left(X_{1i}-\bar{X}_1\right)^2,\ S_{x2}^2=\frac{1}{N-1}\sum_{i=1}^{N}\left(X_{2i}-\bar{X}_2\right)^2,\ \gamma=\frac{1-f}{n},\ f=\frac{n}{N},PRE=\frac{V\left(t_R\right)}{MSE\left(t_i\right)}\ X\ 100,$$

$$C_y^2=\frac{S_y^2}{\bar{Y}^2},\ C_{x_1}^2=\frac{S_{x_1}^2}{\bar{X}_1^2}\,and\ C_{x_2}^2=\frac{S_{x_2}^2}{\bar{X}_2^2}$$

2. MATERIALS AND METHODS

The problem of estimating population mean of the study variable when the population mean of an auxiliary variable(s) is/are known has been discussed among the statisticians in the field of sample survey. Robson [12] developed a product estimator for estimating population mean. Also, Murthy [13] proposed a product type estimator to estimate population mean of study variable by the used of auxiliary information when the coefficient of correlation is negative. Singh and Tailor [14] developed a family of estimators using known values of some parameters to estimate the population mean of the study variable. Abid et al. [15] proposed a class of ratio estimators incorporated non-conventional location parameters for the estimation of population mean. Other researchers are Kadilar and Cingi [16], Koyuncu and Kadilar [17], Yan and Tian [18], Subramani and Kumarapandiyan [19], Yadav et al. [20], Gupta and Yadav [21], Muili et al. [22-25] Audu et al. [26], [27] Muili et al. [28], etc.

Sample mean $\left(\bar{y}\right)$ in simple random sampling without replacement is given as:

$$\bar{y}=\frac{1}{n}\sum_{i=1}^{n}y_i \tag{1.0}$$

$$V\left(\bar{y}\right)=\gamma\bar{Y}^2C_y^2 \tag{1.1}$$

Cochran [1] ratio estimator for estimating population mean $\left(\bar{Y}\right)$ of the study variable $\left(Y\right)$ is given as:

$$t_R=\frac{\bar{y}}{\bar{x}_1}\bar{X}_1 \tag{1.2}$$

$$Bias\left(t_R\right)=\gamma\bar{Y}\left(C_{x_1}^2-\rho_{yx_1}C_yC_x\right) \tag{1.3}$$

$$MSE\left(t_R\right)=\gamma\bar{Y}^2\left(C_y^2+C_{x_1}^2-2\rho_{yx_1}C_yC_{x_1}\right) \tag{1.4}$$

Robson [12] developed a product estimator for estimating population mean $\left(\bar{Y}\right)$ of the study variable $\left(Y\right)$ given as:

$$t_P=\frac{\bar{y}}{\bar{X}_2}\bar{x}_2 \tag{1.5}$$

$$Bias\left(t_P\right)=\gamma\bar{Y}\left(C_{x_1}^2+\rho_{yx_1}C_yC_x\right) \tag{1.6}$$

$$MSE\left(t_P\right)=\gamma\bar{Y}^2\left(C_y^2+C_{x_2}^2+2\rho_{yx_2}C_yC_{x_2}\right) \tag{1.7}$$

Upadhyaya and Singh [2] developed ratio and product estimators for estimation of population mean using known values of coefficient of variation $\left(C_{x_i}\right)$ and coefficient of kurtosis $\left(\beta_2(x_i)\right)$ of variable variables with their biases and mean squares errors (MSEs) given as:

$$t_1=\bar{y}\left(\frac{\bar{X}_1C_{x_1}+\beta_2(x_1)}{\bar{x}_1C_{x_1}+\beta_2(x_1)}\right) \tag{1.8}$$

$$t_2=\bar{y}\left(\frac{\bar{x}_2C_{x_2}+\beta_2(x_2)}{\bar{X}_2C_{x_2}+\beta_2(x_2)}\right) \tag{1.9}$$

$$t_3=\bar{y}\left(\frac{\bar{X}_1\beta_2(x_1)+C_{x_1}}{\bar{x}_1\beta_2(x_1)+C_{x_1}}\right) \tag{1.11}$$

$$t_4=\bar{y}\left(\frac{\bar{x}_2\beta_2(x_2)+C_{x_2}}{\bar{X}_2\beta_2(x_2)+C_{x_2}}\right) \tag{1.12}$$

$$Bias\left(t_1\right)=\gamma\bar{Y}\left(\lambda_1^2C_{x_1}^2-\lambda_1\rho_{yx_1}C_yC_{x_1}\right) \tag{1.13}$$

$$Bias\left(t_2\right)=\gamma\bar{Y}\left(\lambda_2^2C_{x_2}^2+\lambda_2\rho_{yx_2}C_yC_{x_2}\right) \tag{1.14}$$

$$Bias\left(t_3\right)=\gamma\bar{Y}\left(\lambda_3^2C_{x_1}^2-\lambda_3\rho_{yx_1}C_yC_{x_1}\right) \tag{1.15}$$

$$Bias\left(t_4\right)=\gamma\bar{Y}\left(\lambda_4^2C_{x_2}^2+\lambda_4\rho_{yx_2}C_yC_{x_2}\right) \tag{1.16}$$

$$MSE\left(t_1\right)=\gamma\bar{Y}^2\left(C_y^2+\lambda_1^2C_{x_1}^2-2\lambda_1\rho_{yx_1}C_yC_{x_1}\right) \tag{1.17}$$

$$MSE\left(t_2\right)=\gamma\bar{Y}^2\left(C_y^2+\lambda_2^2C_{x_2}^2+2\lambda_2\rho_{yx_2}C_yC_{x_2}\right) \tag{1.18}$$

$$MSE\left(t_3\right)=\gamma\bar{Y}^2\left(C_y^2+\lambda_3^2C_{x_1}^2-2\lambda_3\rho_{yx_1}C_yC_{x_1}\right) \tag{1.19}$$

$$MSE\left(t_4\right)=\gamma\bar{Y}^2\left(C_y^2+\lambda_4^2C_{x_2}^2+2\lambda_4\rho_{yx_2}C_yC_{x_2}\right) \tag{1.21}$$

where $\lambda_1 = \frac{\bar{X}_1 C_{x_1}}{\bar{X}_1 C_{x_1} + \beta_2(x_1)}$, $\lambda_2 = \frac{\bar{X}_2 C_{x_2}}{\bar{X}_2 C_{x_2} + \beta_2(x_2)}$ $\lambda_3 = \frac{\bar{X}_1 \beta_2(x_1)}{\bar{X}_1 \beta_2(x_1) + C_{x_1}}$, $\lambda_4 = \frac{\bar{X}_2 \beta_2(x_2)}{\bar{X}_2 \beta_2(x_2) + C_{x_2}}$

Singh [29] proposed a ratio-cum-product estimator of population mean using the two auxiliary variables as:

$$t_5 = \bar{y}\left(\frac{\bar{X}_1}{\bar{x}_1}\right)\left(\frac{\bar{x}_2}{\bar{X}_2}\right) \tag{1.22}$$

$$Bias\left(t_5\right) = \gamma\bar{Y}\left(C_{x_1}^2\left(1-\kappa_{yx_1}\right) + C_{x_2}^2\left(\kappa_{yx_2} - \kappa_{x_1x_2}\right)\right) \tag{1.23}$$

$$MSE\left(t_5\right) = \gamma\bar{Y}^2\left(C_y^2 + C_{x_1}^2\left(1-2\kappa_{yx_1}\right) + C_{x_2}^2\left\{1+2(\kappa_{yx_2} - \kappa_{x_1x_2})\right\}\right) \tag{1.24}$$

Singh and Tailor [30] also developed a ratio-cum-product estimator for estimation of population mean incorporated coefficient of variation between auxiliary variables into the work of Singh (1967) as:

$$t_6 = \bar{y}\left(\frac{\bar{X}_1 + \rho_{x_1x_2}}{\bar{x}_1 + \rho_{x_1x_2}}\right)\left(\frac{\bar{x}_2 + \rho_{x_1x_2}}{\bar{X}_2 + \rho_{x_1x_2}}\right) \tag{1.25}$$

$$Bias\left(t_6\right) = \gamma\bar{Y}\left(\mu_1 C_{x_1}^2\left(\mu_1 - \kappa_{yx_1}\right) + \mu_2 C_{x_2}^2\left(\mu_2\kappa_{yx_2} - \mu_1\kappa_{x_1x_2}\right)\right) \tag{1.26}$$

$$MSE\left(t_6\right) = \gamma\bar{Y}^2\left(C_y^2 + \mu_1 C_{x_1}^2\left(\mu_1 - 2\kappa_{yx_1}\right) + \mu_2 C_{x_2}^2\left\{\mu_2 + 2(\kappa_{yx_2} - \mu_1\kappa_{x_1x_2})\right\}\right) \tag{1.27}$$

Where $\mu_1 = \frac{\bar{X}_1}{\bar{X}_1 + \rho_{x_1x_2}}$, *and* $\mu_2 = \frac{\bar{X}_2}{\bar{X}_2 + \rho_{x_1x_2}}$

Tailor *et al*. [31] proposed two ratio-cum-product estimators of population mean using both coefficient of variation and coefficient of kurtosis of auxiliary variables as:

$$t_7 = \bar{y}\left(\frac{\bar{X}_1 C_{x_1} + \beta_2(x_1)}{\bar{x}_1 C_{x_1} + \beta_2(x_1)}\right)\left(\frac{\bar{x}_2 C_{x_2} + \beta_2(x_2)}{\bar{X}_2 C_{x_2} + \beta_2(x_2)}\right) \tag{1.28}$$

$$t_8 = \bar{y}\left(\frac{\bar{X}_1 \beta_2(x_1) + C_{x_1}}{\bar{x}_1 \beta_2(x_1) + C_{x_1}}\right)\left(\frac{\bar{x}_2 \beta_2(x_2) + C_{x_2}}{\bar{X}_2 \beta_2(x_2) + C_{x_2}}\right) \tag{1.29}$$

$$Bias\left(t_7\right) = \gamma\bar{Y}\left(\lambda_1 C_{x_1}^2\left(\lambda_1 - \kappa_{yx_1}\right) + \lambda_2 C_{x_2}^2\left(\lambda_2\kappa_{yx_2} - \lambda_1\kappa_{x_1x_2}\right)\right) \tag{1.31}$$

$$MSE\left(t_7\right) = \gamma\bar{Y}^2\left(C_y^2 + \lambda_1 C_{x_1}^2\left(\lambda_1 - 2\kappa_{yx_1}\right) + \lambda_2 C_{x_2}^2\left\{\lambda_2 + 2(\kappa_{yx_2} - \lambda_1\kappa_{x_1x_2})\right\}\right) \tag{1.32}$$

$$Bias\left(t_8\right) = \gamma\bar{Y}\left(\lambda_3 C_{x_1}^2\left(\lambda_3 - \kappa_{yx_1}\right) + \lambda_4 C_{x_2}^2\left(\lambda_4\kappa_{yx_2} - \lambda_3\kappa_{x_1x_2}\right)\right) \tag{1.33}$$

$$MSE\left(t_8\right)=\gamma\bar{Y}^2\left(C_y^2+\lambda_3C_{x_1}^2\left(\lambda_3-2\kappa_{yx_1}\right)+\lambda_4C_{x_2}^2\left\{\lambda_4+2(\kappa_{yx_2}-\lambda_3\kappa_{x_1x_2})\right\}\right) \tag{1.34}$$

Yadav et al. [32] developed a ratio-cum-product for estimation of population mean using known values of coefficient of kurtosis and median of auxiliary variables as:

$$t_9=\bar{y}\left(\frac{\bar{X}_1\beta_2(x_1)+M_d(x_1)}{\bar{x}_1\beta_2(x_1)+M_d(x_1)}\right)\left(\frac{\bar{x}_2\beta_2(x_2)+M_d(x_2)}{\bar{X}_2\beta_2(x_2)+M_d(x_2)}\right) \tag{1.35}$$

$$Bias\left(t_9\right)=\gamma\bar{Y}\left(\eta_1C_{x_1}^2\left(\eta_1-\kappa_{yx_1}\right)+\eta_2C_{x_2}^2\left(\eta_2\kappa_{yx_2}-\eta_1\kappa_{x_1x_2}\right)\right) \tag{1.36}$$

$$MSE\left(t_9\right)=\gamma\bar{Y}^2\left(C_y^2+\eta_1C_{x_1}^2\left(\eta_1-2\kappa_{yx_1}\right)+\eta_2C_{x_2}^2\left\{\eta_2+2(\kappa_{yx_2}-\eta_1\kappa_{x_1x_2})\right\}\right) \tag{1.37}$$

Where $\eta_1=\dfrac{\bar{X}_1\beta_2(x_1)}{\bar{X}_1\beta_2(x_1)+M_d(x_1)}$, $\eta_2=\dfrac{\bar{X}_2\beta_2(x_2)}{\bar{X}_2\beta_2(x_2)+M_d(x_2)}$

2.1 Proposed Estimator

Having studied the works of Singh and Tailor [30], Tailor *et al.* [31] and Yadav et al. [32], we proposed a family of ratio-cum-product estimators for estimating population mean using information of percentiles of auxiliary variables as:

$$t_{p1}=\bar{y}\left(\frac{\bar{X}_1\beta_2(x_1)+P_{55}(x_1)}{\bar{x}_1\beta_2(x_1)+P_{55}(x_1)}\right)\left(\frac{\bar{x}_2\beta_2(x_2)+P_{55}(x_2)}{\bar{X}_2\beta_2(x_2)+P_{55}(x_2)}\right) \tag{2.1}$$

$$t_{p2}=\bar{y}\left(\frac{\bar{X}_1\beta_2(x_1)+P_{60}(x_1)}{\bar{x}_1\beta_2(x_1)+P_{60}(x_1)}\right)\left(\frac{\bar{x}_2\beta_2(x_2)+P_{60}(x_2)}{\bar{X}_2\beta_2(x_2)+P_{60}(x_2)}\right) \tag{2.2}$$

$$t_{p3}=\bar{y}\left(\frac{\bar{X}_1\beta_2(x_1)+P_{65}(x_1)}{\bar{x}_1\beta_2(x_1)+P_{65}(x_1)}\right)\left(\frac{\bar{x}_2\beta_2(x_2)+P_{65}(x_2)}{\bar{X}_2\beta_2(x_2)+P_{65}(x_2)}\right) \tag{2.3}$$

$$t_{p4}=\bar{y}\left(\frac{\bar{X}_1\beta_2(x_1)+P_{70}(x_1)}{\bar{x}_1\beta_2(x_1)+P_{70}(x_1)}\right)\left(\frac{\bar{x}_2\beta_2(x_2)+P_{70}(x_2)}{\bar{X}_2\beta_2(x_2)+P_{70}(x_2)}\right) \tag{2.4}$$

$$t_{p5}=\bar{y}\left(\frac{\bar{X}_1\beta_2(x_1)+P_{75}(x_1)}{\bar{x}_1\beta_2(x_1)+P_{75}(x_1)}\right)\left(\frac{\bar{x}_2\beta_2(x_2)+P_{75}(x_2)}{\bar{X}_2\beta_2(x_2)+P_{75}(x_2)}\right) \tag{2.5}$$

$$t_{p6}=\bar{y}\left(\frac{\bar{X}_1\beta_2(x_1)+P_{80}(x_1)}{\bar{x}_1\beta_2(x_1)+P_{80}(x_1)}\right)\left(\frac{\bar{x}_2\beta_2(x_2)+P_{80}(x_2)}{\bar{X}_2\beta_2(x_2)+P_{80}(x_2)}\right) \tag{2.6}$$

$$t_{p7}=\bar{y}\left(\frac{\bar{X}_1\beta_2(x_1)+P_{85}(x_1)}{\bar{x}_1\beta_2(x_1)+P_{85}(x_1)}\right)\left(\frac{\bar{x}_2\beta_2(x_2)+P_{85}(x_2)}{\bar{X}_2\beta_2(x_2)+P_{85}(x_2)}\right) \tag{2.7}$$

$$t_{p8}=\bar{y}\left(\frac{\bar{X}_1\beta_2(x_1)+P_{90}(x_1)}{\bar{x}_1\beta_2(x_1)+P_{90}(x_1)}\right)\left(\frac{\bar{x}_2\beta_2(x_2)+P_{90}(x_2)}{\bar{X}_2\beta_2(x_2)+P_{90}(x_2)}\right) \tag{2.8}$$

$$t_{p9}=\bar{y}\left(\frac{\bar{X}_1\beta_2(x_1)+P_{95}(x_1)}{\bar{x}_1\beta_2(x_1)+P_{95}(x_1)}\right)\left(\frac{\bar{x}_2\beta_2(x_2)+P_{95}(x_2)}{\bar{X}_2\beta_2(x_2)+P_{95}(x_2)}\right) \tag{2.9}$$

$$t_{p10}=\bar{y}\left(\frac{\bar{X}_1\beta_2(x_1)+P_{99}(x_1)}{\bar{x}_1\beta_2(x_1)+P_{99}(x_1)}\right)\left(\frac{\bar{x}_2\beta_2(x_2)+P_{99}(x_2)}{\bar{X}_2\beta_2(x_2)+P_{99}(x_2)}\right) \tag{2.11}$$

The proposed estimators can be written in a general form as:

$$t_{pi}=\bar{y}\left(\frac{\bar{X}_1\beta_2(x_1)+P_k(x_1)}{\bar{x}_1\beta_2(x_1)+P_k(x_1)}\right)\left(\frac{\bar{x}_2\beta_2(x_2)+P_k(x_2)}{\bar{X}_2\beta_2(x_2)+P_k(x_2)}\right) \tag{2.12}$$

where

$$i=1,2,...,10 \qquad k=55,60,...,99$$

2.1.1 Properties (bias and MSE) of the proposed estimators

To obtain the bias and MSE, we define $e_0=\dfrac{\bar{y}-\bar{Y}}{\bar{Y}}$, $e_1=\dfrac{\bar{x}_1-\bar{X}_1}{\bar{X}_1}$ and $e_2=\dfrac{\bar{x}_2-\bar{X}_2}{\bar{X}_2}$ such that $\bar{y}=\bar{Y}\left(1+e_0\right)$, $\bar{x}_1=\bar{X}_1\left(1+e_1\right) and\ \bar{x}_2=\bar{X}_2\left(1+e_2\right)$

, from the definitions of e_0, e_1 and e_2, we obtain

$$\left.\begin{array}{l} E(e_0)=E(e_1)=E(e_2)=0,\ E(e_0^2)=\gamma C_y^2 \\ E(e_1^2)=\gamma C_{x_1}^2,\ E(e_2^2)=\gamma C_{x_2}^2, E(e_0e_1)=\gamma\rho_{yx_1}C_yC_{x_1} \\ E(e_0e_2)=\gamma\rho_{yx_2}C_yC_{x_2},\ E(e_1e_2)=\gamma\rho_{x_1x_2}C_{x_1}C_{x_2}, \end{array}\right\} \quad (2.13)$$

Expressing (2.12) in terms of e_0, e_1 and e_2, we have

$$t_{pi}=\bar{Y}(1+e_0)\left(\frac{\bar{X}_1\beta_2(x_1)+P_k(x_1)}{\bar{X}_1(1+e_1)\beta_2(x_1)+P_k(x_1)}\right)\left(\frac{\bar{X}_2(1+e_2)\beta_2(x_2)+P_k(x_2)}{\bar{X}_2\beta_2(x_2)+P_k(x_2)}\right) \quad (2.14)$$

$$t_{pi}=\bar{Y}(1+e_0)(1+\phi_1e_1)^{-1}(1+\phi_2e_2) \quad (2.15)$$

where $\phi_1=\dfrac{\bar{X}_1\beta_2(x_1)}{\bar{X}_1\beta_2(x_1)+P_k(x_1)},\ \phi_2=\dfrac{\bar{X}_2\beta_2(x_2)}{\bar{X}_2\beta_2(x_2)+P_k(x_2)}$

Simplifying (2.15) up to first order approximation, it reduces to (2.16) as:

$$t_{pi}=\bar{Y}\left(1+e_0-\phi_1e_1-\phi_1e_0e_1+\phi_1^2e_1^2+\phi_2e_2+\phi_2e_0e_2-\phi_1\phi_2e_1e_2\right) \quad (2.16)$$

Subtracting $\bar{Y}$ from both sides

$$\left(t_{pi}-\bar{Y}\right)=\bar{Y}+\bar{Y}\left(e_0-\phi_1e_1-\phi_1e_0e_1+\phi_1^2e_1^2+\phi_2e_2+\phi_2e_0e_2-\phi_1\phi_2e_1e_2\right)-\bar{Y} \quad (2.17)$$

Taking expectation of both sides

$$E\left(t_{pi}-\bar{Y}\right)=\bar{Y}E\left(e_0-\phi_1e_1-\phi_1e_0e_1+\phi_1^2e_1^2+\phi_2e_2+\phi_2e_0e_2-\phi_1\phi_2e_1e_2\right) \quad (2.18)$$

Applying the results of (2.13) to (2.18), gives the bias as:

$$Bias\left(t_{pi}\right)=\gamma\bar{Y}\left(\phi_{1_i}^2C_{x_1}^2-\phi_{1_i}\rho_{yx_1}C_yC_{x_1}+\phi_{2_i}\rho_{yx_2}C_yC_{x_2}-\phi_{1_i}\phi_{2_i}\rho_{x_1x_2}C_{x_1}C_{x_2}\right) \quad (2.19)$$

$$Bias\left(t_{pi}\right)=\gamma\bar{Y}\left(\phi_{1_i}C_{x_1}^2\left(\phi_{1_i}-\kappa_{yx_1}\right)+\phi_{2_i}C_{x_2}^2\left(\phi_{2_i}\kappa_{yx_2}-\phi_{1_i}\kappa_{x_1x_2}\right)\right),\quad i=1,2,...,10 \quad (2.21)$$

where $\kappa_{yx_1}=\rho_{yx_1}\left(\dfrac{C_y}{C_{x_1}}\right),\ \kappa_{yx_2}=\rho_{yx_2}\left(\dfrac{C_y}{C_{x_2}}\right),\ \kappa_{x_1x_2}=\rho_{x_1x_2}\left(\dfrac{C_{x_1}}{C_{x_2}}\right)$

Squaring and taking expectation of (2.18), gives

$$MSE\left(t_{pi}\right)=\left(\bar{Y}E\left(e_0-\phi_{1_i}e_1+\phi_{2_i}e_2\right)\right)^2 \quad (2.22)$$

Expanding (2.22)

$$MSE\left(t_{pi}\right)=\bar{Y}^2E\left(e_0^2+\phi_{1_i}^2e_1^2+\phi_{2_i}^2e_2^2-2\phi_{1_i}e_0e_1+2\phi_{2_i}e_0e_2-2\phi_{1_i}\phi_{2_i}e_1e_2\right) \quad (2.23)$$

Applying the results of (2.13) to (2.23), gives

$$MSE\left(t_{pi}\right)=\gamma\bar{Y}^{2}\left(C_{y}^{2}+\phi_{1_i}^{2}C_{x_1}^{2}+\phi_{2_i}^{2}C_{x_1}^{2}-2\phi_{1_i}\rho_{yx_1}C_{y}C_{x_1}+2\phi_{2_i}\rho_{yx_2}C_{y}C_{x_2}-2\phi_{1_i}\phi_{2_i}\rho_{x_1x_2}C_{x_1}C_{x_2}\right) \quad (2.24)$$

$$MSE\left(t_{pi}\right)=\gamma\bar{Y}^{2}\left(C_{y}^{2}+\phi_{1_i}C_{x_1}^{2}\left(\phi_{1_i}-2\kappa_{yx_1}\right)+\phi_{2_i}C_{x_2}^{2}\left\{\phi_{2_i}+2\left(\kappa_{yx_2}-\phi_{1_i}\kappa_{x_1x_2}\right)\right\}\right),\quad i=1,2,\ldots,10 \quad (2.25)$$

3. EFFICIENCY COMPARISON

Efficiencies of the proposed estimators are compared with efficiencies of the existing estimators in the study

The proposed estimators $\left(t_{pi}\right)$ of the finite population mean are more efficient than $\bar{y}$ if,

$$MSE\left(t_{pi}\right)<V\left(\bar{y}\right) \qquad i=1,2,\ldots,10$$

$$\gamma\bar{Y}^{2}\left(C_{y}^{2}+\phi_{1_i}C_{x_1}^{2}\left(\phi_{1_i}-2\kappa_{yx_1}\right)+\phi_{2_i}C_{x_2}^{2}\left\{\phi_{2_i}+2\left(\kappa_{yx_2}-\phi_{1_i}\kappa_{x_1x_2}\right)\right\}\right)<\gamma\bar{Y}^{2}C_{y}^{2} \quad (3.1)$$

The proposed estimators $\left(t_{pi}\right)$ of the finite population mean are more efficient than t_R if,

$$MSE\left(t_{pi}\right)<MSE\left(t_{R}\right) \qquad i=1,2,\ldots,10$$

$$\gamma\bar{Y}^{2}\left(C_{y}^{2}+\phi_{1_i}C_{x_1}^{2}\left(\phi_{1_i}-2\kappa_{yx_1}\right)+\phi_{2_i}C_{x_2}^{2}\left\{\phi_{2_i}+2\left(\kappa_{yx_2}-\phi_{1_i}\kappa_{x_1x_2}\right)\right\}\right)<\gamma\bar{Y}^{2}\left(C_{y}^{2}+C_{x_1}^{2}-2\rho_{yx_1}C_{y}C_{x_1}\right) \quad (3.2)$$

The proposed estimators $\left(t_{pi}\right)$ of the finite population mean are more efficient than t_P if,

$$MSE\left(t_{pi}\right)<MSE\left(t_{P}\right) \qquad i=1,2,\ldots,10$$

$$\gamma\bar{Y}^{2}\left(C_{y}^{2}+\phi_{1_i}C_{x_1}^{2}\left(\phi_{1_i}-2\kappa_{yx_1}\right)+\phi_{2_i}C_{x_2}^{2}\left\{\phi_{2_i}+2\left(\kappa_{yx_2}-\phi_{1_i}\kappa_{x_1x_2}\right)\right\}\right)<\gamma\bar{Y}^{2}\left(C_{y}^{2}+C_{x_2}^{2}+2\rho_{yx_2}C_{y}C_{x_2}\right) \quad (3.3)$$

The proposed estimators $\left(t_{pi}\right)$ of the finite population mean are more efficient than t_j if,

$$MSE\left(t_{pi}\right)<MSE\left(t_{j}\right) \qquad i=1,2,\ldots,10 \qquad j=1,2,3,4$$

$$\left(C_{y}^{2}+\phi_{1_i}C_{x_1}^{2}\left(\phi_{1_i}-2\kappa_{yx_1}\right)+\phi_{2_i}C_{x_2}^{2}\left\{\phi_{2_i}+2\left(\kappa_{yx_2}-\phi_{1_i}\kappa_{x_1x_2}\right)\right\}\right)<\left(C_{y}^{2}+\lambda_{j}^{2}C_{x_1}^{2}-2\lambda_{j}\rho_{yx_1}C_{y}C_{x_1}\right) \quad (3.4)$$

The proposed estimators $\left(t_{pi}\right)$ of the finite population mean are more efficient than t_5 if,

$$MSE\left(t_{pi}\right)<MSE\left(t_{5}\right) \qquad i=1,2,\ldots,10$$

$$\left(C_{y}^{2}+\phi_{1_i}C_{x_1}^{2}\left(\phi_{1_i}-2\kappa_{yx_1}\right)+\phi_{2_i}C_{x_2}^{2}\left\{\phi_{2_i}+2\left(\kappa_{yx_2}-\phi_{1_i}\kappa_{x_1x_2}\right)\right\}\right)<\left(C_{y}^{2}+C_{x_1}^{2}\left(1-2\kappa_{yx_1}\right)+C_{x_2}^{2}\left\{1+2(\kappa_{yx_2}-\kappa_{x_1x_2})\right\}\right) \quad (3.5)$$

The proposed estimators $\left(t_{pi}\right)$ of the finite population mean are more efficient than t_6 if,

$$MSE\left(t_{pi}\right) < MSE\left(t_6\right) \qquad i = 1, 2, ..., 10$$

$$\left(\phi_{1_i} C_{x_1}^2 \left(\phi_{1_i} - 2\kappa_{yx_1}\right) + \phi_{2_i} C_{x_2}^2 \left\{\phi_{2_i} + 2\left(\kappa_{yx_2} - \phi_{1_i}\kappa_{x_1x_2}\right)\right\}\right) < \left(\mu_1 C_{x_1}^2 \left(\mu_1 - 2\kappa_{yx_1}\right) + \mu_2 C_{x_2}^2 \left\{\mu_2 + 2(\kappa_{yx_2} - \mu_1\kappa_{x_1x_2})\right\}\right) \quad (3.6)$$

The proposed estimators $\left(t_{pi}\right)$ of the finite population mean are more efficient than t_7 if,

$$MSE\left(t_{pi}\right) < MSE\left(t_7\right) \qquad i = 1, 2, ..., 10$$

$$\left(\phi_{1_i} C_{x_1}^2 \left(\phi_{1_i} - 2\kappa_{yx_1}\right) + \phi_{2_i} C_{x_2}^2 \left\{\phi_{2_i} + 2\left(\kappa_{yx_2} - \phi_{1_i}\kappa_{x_1x_2}\right)\right\}\right) < \left(\lambda_1 C_{x_1}^2 \left(\lambda_1 - 2\kappa_{yx_1}\right) + \lambda_2 C_{x_2}^2 \left\{\lambda_2 + 2(\kappa_{yx_2} - \lambda_1\kappa_{x_1x_2})\right\}\right) \quad (3.7)$$

The proposed estimators $\left(t_{pi}\right)$ of the finite population mean are more efficient than t_8 if,

$$MSE\left(t_{pi}\right) < MSE\left(t_8\right) \qquad i = 1, 2, ..., 10$$

$$\left(\phi_{1_i} C_{x_1}^2 \left(\phi_{1_i} - 2\kappa_{yx_1}\right) + \phi_{2_i} C_{x_2}^2 \left\{\phi_{2_i} + 2\left(\kappa_{yx_2} - \phi_{1_i}\kappa_{x_1x_2}\right)\right\}\right) < \left(\lambda_3 C_{x_1}^2 \left(\lambda_3 - 2\kappa_{yx_1}\right) + \lambda_4 C_{x_2}^2 \left\{\lambda_4 + 2(\kappa_{yx_2} - \lambda_3\kappa_{x_1x_2})\right\}\right) \quad (3.8)$$

The proposed estimators $\left(t_{pi}\right)$ of the finite population mean are more efficient than t_9 if,

$$MSE\left(t_{pi}\right) < MSE\left(t_9\right) \qquad i = 1, 2, ..., 10$$

$$\left(\phi_{1_i} C_{x_1}^2 \left(\phi_{1_i} - 2\kappa_{yx_1}\right) + \phi_{2_i} C_{x_2}^2 \left\{\phi_{2_i} + 2\left(\kappa_{yx_2} - \phi_{1_i}\kappa_{x_1x_2}\right)\right\}\right) < \left(\eta_1 C_{x_1}^2 \left(\eta_1 - 2\kappa_{yx_1}\right) + \eta_2 C_{x_2}^2 \left\{\eta_2 + 2(\kappa_{yx_2} - \eta_1\kappa_{x_1x_2})\right\}\right) \quad (3.9)$$

When conditions (3.1), (3.2), (3.3), (3.4), (3.5), (3.6), (3.7), (3.8) and (3.9) are satisfied, we can conclude that the proposed estimators are more efficient than sample mean, the ratio estimator, product estimator, and other existing estimators considered in the study.

3.1 Empirical Study

In order to access the performance of the proposed estimators, we considered a real population as given in Yadav et al [32].

$$N = 30, n = 10, \bar{Y} = 17.5,\ \bar{X}_1 = 47.1333,\ \bar{X}_2 = 4.4637,\ \rho_{yx_1} = 0.3637,\ \rho_{yx_2} = -0.1994,\ \rho_{x_1x_2} = 0.0736,$$
$$\beta_2(x_1) = 0.06206,\ \beta_2(x_2) = 0.2296,\ C_y = 0.4758,\ C_{x_1} = 0.6046,\ C_{x_2} = 0.8727,\ M_d(x_1) = 36,$$
$$M_d(x_2) = 2.21,\ P_{99x_1} = 125, P_{99x_2} = 13.12,\ P_{95x_1} = 110.15,\ P_{95x_2} = 12.89,\ P_{90x_1} = 96.9,\ P_{90x_2} = 12.24,$$
$$P_{85x_1} = 95.35,\ P_{85x_2} = 11.64,\ P_{80x_1} = 67.8,\ P_{80x_2} = 8.2,\ P_{75x_1} = 59.25,\ P_{75x_2} = 6.88, P_{70x_1} = 53.9, P_{70x_2} = 3.84,$$
$$P_{65x_1} = 49,\ P_{65x_2} = 2.61,\ P_{60x_1} = 46.8,\ P_{60x_2} = 2.49,\ P_{55x_1} = 40.25,\ P_{55x_2} = 2.36.$$

Table 1. MSE and PRE of existing and proposed estimators

Estimator	MSE	PRE
Sample Mean $\left(\bar{y}\right)$	4.6129	100
Ratio Estimator $\left(t_R\right)$	7.7989	59.14808
Product Estimator $\left(t_P\right)$	16.7563	27.52935
$\left(t_1\right)$ [2]	7.5756	60.89155
$\left(t_2\right)$ [2]	15.2646	30.21959
$\left(t_3\right)$ [2]	7.5865	60.80406
$\left(t_4\right)$ [2]	7.3175	63.03929
$\left(t_5\right)$ [29]	18.3606	25.12391
$\left(t_6\right)$ [30]	17.9278	25.73043
$\left(t_7\right)$ [31]	16.7655	27.51424
$\left(t_8\right)$ [31]	9.4541	48.79259
Yadav *et al*. (2016) $\left(t_9\right)$	4.4003	104.8315
Proposed Estimator $\left(t_{p1}\right)$	4.3457	106.1486
Proposed Estimator $\left(t_{p2}\right)$	4.2430	108.7179
Proposed Estimator $\left(t_{p3}\right)$	4.1837	110.2589
Proposed Estimator $\left(t_{p4}\right)$	3.9026	118.2007
Proposed Estimator $\left(t_{p5}\right)$	3.7804	122.0215
Proposed Estimator $\left(t_{p6}\right)$	3.7756	122.1766
Proposed Estimator $\left(t_{p7}\right)$	3.8285	120.4884
Proposed Estimator $\left(t_{p8}\right)$	3.8359	120.256
Proposed Estimator $\left(t_{p9}\right)$	3.8650	119.3506
Proposed Estimator $\left(t_{p10}\right)$	3.8952	118.4252

4. RESULTS AND DISCUSSION

A family of ratio-cum-product estimators for estimation of population mean of the study variable using known population parameters of two auxiliary variables. The bias and mean square error (MSE) of the proposed estimators were derived up to first order of appreciation. Theoretical comparison of the proposed ratio-cum-product estimators of population mean with sample mean $\left(\bar{y}\right)$, ratio estimator $\left(t_R\right)$, product estimator $\left(t_P\right)$ and other existing estimators considered in the study were established. The mean square errors (MSEs) of the proposed estimators are lesser than sample mean, ratio estimator, product estimator and other estimators considered in the study. The performance of the proposed estimators over the sample mean, ratio estimator, product estimator and other selected existing estimators using a real population were obtained.

5. CONCLUSION

The study proposed a family of new ratio-cum-product estimators of finite population mean based on the information obtained from the percentiles of auxiliary variables. The results in Table 1 clearly showed that the proposed ratio-cum-product estimators performed better than the sample mean, ratio estimator, product estimator and other existing estimators considered in the study having minimum Mean Square Error (MSE) and the highest Percentage Relative Error (PRE).

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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