

Research Article

Characteristic Properties of Type-2 Smarandache Ruled Surfaces According to the Type-2 Bishop Frame in E^3

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In this paper, we define and investigate a special kind of ruled surfaces called type-2 Smarandache ruled surfaces related to the type-2 Bishop frame in E^3 . From this point and depending on the type-2 Bishop curvature, we provide the necessary and sufficient conditions that allow these surfaces to be developable in a minimal amount of time. Furthermore, an example is given to clear the results.

1. Introduction

In the classical differential geometry, the theory of ruled surfaces is one of its branches which has been developed by several researchers. A ruled surface is generally defined as the set of a family of straight lines that depend on a parameter that is mentioned as the ruled surface's rulings. A ruled surface's parametric representation is $Y(\sigma, v) = c(\sigma) + vX(\sigma)$ where $c(\sigma)$ is the base curve of $Y(\sigma, v)$ and $X(\sigma)$ define the ruling directions [1, 2]. Surfaces' developability and minimalist notions are two of their most important properties. One of the most interesting points is the study of ruled surfaces with different moving frames, as seen in this example [3–7].

The Smarandache curve in Euclidean and Minkowski spaces is the curve whose position vector is made by Frenet frame vectors on another regular curve [8–11]. Several researchers [12–20] have recently studied Smarandache curves in Minkowski and the Euclidean spaces.

In this work, in E^3 , we introduce the definitions of type-2 Smarandache ruled surfaces using the type-2 Bishop frame, namely, $\mu_1\mu_2$, μ_1B , and μ_2B type-2 Smarandache ruled surfaces. Our main results are presented in theorems that look into the necessary and sufficient conditions for those surfaces to be developable and minimal. Throughout the response, an example with illustrations is created.

2. Preliminaries

Let E^3 be a 3-dimensional Euclidean space provided with the metric

$$\langle \cdot, \cdot \rangle = du_1^2 + du_2^2 + du_3^2, \quad (1)$$

where (u_1, u_2, u_3) is the rectangular coordinate system of E^3 .

Representing the moving Frenet frame along its regular curve ψ by $\{T, N, B\}$ in conjunction with curvature functions κ and τ in E^3 , the Frenet formula is given as follows [1]:

$$\frac{d}{d\sigma} \begin{pmatrix} T(\sigma) \\ N(\sigma) \\ B(\sigma) \end{pmatrix} = \begin{pmatrix} 0 & \kappa(\sigma) & 0 \\ -\kappa(\sigma) & 0 & \tau(\sigma) \\ 0 & -\tau(\sigma) & 0 \end{pmatrix} \begin{pmatrix} T(\sigma) \\ N(\sigma) \\ B(\sigma) \end{pmatrix}, \quad (2)$$

where $\langle T, T \rangle = \langle N, N \rangle = \langle B, B \rangle = 1$ and $\langle T, N \rangle = \langle T, B \rangle = \langle N, B \rangle = 0$.

For any arbitrary curve ψ with $\tau \neq 0$ in \mathbf{E}^3 , the type-2 Bishop frame of ψ is given as follows [21]:

$$\frac{d}{d\sigma} \begin{pmatrix} \mu_1(\sigma) \\ \mu_2(\sigma) \\ B(\sigma) \end{pmatrix} = \begin{pmatrix} 0 & 0 & -k_1(\sigma) \\ 0 & 0 & -k_2(\sigma) \\ k_1(\sigma) & k_2(\sigma) & 0 \end{pmatrix} \begin{pmatrix} \mu_1(\sigma) \\ \mu_2(\sigma) \\ B(\sigma) \end{pmatrix}, \quad (3)$$

where k_1 and k_2 are the type-2 Bishop curvatures and satisfying

$$\begin{pmatrix} T(\sigma) \\ N(\sigma) \\ B(\sigma) \end{pmatrix} = \begin{pmatrix} \sin \theta(\sigma) & -\cos \theta(\sigma) & 0 \\ \cos \theta(\sigma) & \sin \theta(\sigma) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mu_1(\sigma) \\ \mu_2(\sigma) \\ B(\sigma) \end{pmatrix}, \quad (4)$$

where $\theta(\sigma) = \arctan(k_2/k_1)$ and

$$\begin{aligned} k_1 &= -\tau \cos \theta(\sigma), \\ k_2 &= -\tau \sin \theta(\sigma). \end{aligned} \quad (5)$$

Definition 1. [21]. $\mu_1\mu_2$ type-2 Smarandache curves of the curve $\psi(\sigma)$ via $\{\mu_1, \mu_2, B\}$ are given as

$$\beta(\sigma^*(\sigma)) = \frac{1}{\sqrt{2}}(\mu_1(\sigma) + \mu_2(\sigma)). \quad (6)$$

Definition 2. [21]. μ_1B type-2 Smarandache curves of the curve $\psi(\sigma)$ via $\{\mu_1, \mu_2, B\}$ are given as

$$\gamma(\sigma^*(\sigma)) = \frac{1}{\sqrt{2}}(\mu_1(\sigma) + B(\sigma)). \quad (7)$$

Definition 3. [21]. μ_2B type-2 Smarandache curves of the curve $\psi(\sigma)$ via $\{\mu_1, \mu_2, B\}$ are given as

$$\delta(\sigma^*(\sigma)) = \frac{1}{\sqrt{2}}(\mu_2(\sigma) + B(\sigma)). \quad (8)$$

A ruled surface v in \mathbf{E}^3 can be reparametrized as

$$v(\sigma, v) = \psi(\sigma) + v\chi(\sigma), \quad (9)$$

where $\psi(\sigma)$ is really the base curve and $\chi(\sigma)$ is its unit which defines a space curve that characterizes the straight line's direction [22].

v 's unit normal vector N is given as follows [23]:

$$\mathbf{U} = \frac{v_\sigma \times v_v}{\|v_\sigma \times v_v\|}, \quad (10)$$

where $v_\sigma = \partial v / \partial \sigma$ and $v_v = \partial v / \partial v$. The Gaussian curvature K and the mean curvature H are given as follows [23]:

$$\begin{aligned} K &= \frac{\ell n - m^2}{EG - F^2}, \\ H &= \frac{En + G\ell - 2mF}{2(EG - F^2)}, \end{aligned} \quad (11)$$

where $E = \|Y_\sigma\|^2$, $F = \langle Y_\sigma, Y_v \rangle$, $G = \|Y_v\|^2$, $\ell = \langle Y_{\sigma\sigma}, \mathbf{U} \rangle$, $m = \langle Y_{\sigma v}, \mathbf{U} \rangle$, and $n = \langle Y_{vv}, \mathbf{U} \rangle$. The normal curvature, geodesic curvature, and geodesic torsion that connects the curve $\psi(\sigma)$ on Y are computed as follows:

$$\kappa_n = \langle \psi'', \mathbf{U} \rangle, \kappa_g = \langle \mathbf{U} \times T, T' \rangle, \tau_g = \langle \mathbf{U} \times \mathbf{U}', T' \rangle. \quad (12)$$

Definition 4. A ruled surface is developable if and only if $K = 0$ and minimal if and only if $H = 0$.

3. Main Results

In this part, we define the type-2 Smarandache ruled surfaces within Euclidean 3-space \mathbf{E}^3 referring to the frame $\{\mu_1, \mu_2, B\}$. Furthermore, we evaluate the sufficient and necessary conditions that enable these surfaces to be developable and minimal.

3.1. $\mu_1\mu_2$ Type-2 Smarandache Ruled Surface

Definition 5. For a regular curve $\psi = \psi(\sigma)$ in \mathbf{E}^3 related to the frame $\{\mu_1, \mu_2, B\}$, the $\mu_1\mu_2$ type-2 Smarandache ruled surface is given as

$$\Omega = \Omega(\sigma, v) = \frac{1}{\sqrt{2}}(\mu_1(\sigma) + \mu_2(\sigma)) + vB(\sigma). \quad (13)$$

Theorem 6. Let $\Omega = \Omega(\sigma, v)$ be the $\mu_1\mu_2$ type-2 Smarandache ruled surface in \mathbf{E}^3 defined by (13). Then, we have

- (1) Ω is a developable surface with asymptotic base curve $\psi(\sigma)$
- (2) Ω is a minimal surface if and only if the type-2 Bishop curvatures satisfy the following equation

$$k_1 = k_2 e^{\sigma+c}, \quad (14)$$

where c is real constant.

Proof. Considering that the $\mu_1\mu_2$ type-2 Smarandache ruled surface given by (13), then, the velocity vectors of Ω are given as follows:

$$\begin{aligned} \Omega_\sigma &= vk_1\mu_1 + vk_2\mu_2 - \left(\frac{k_1 + k_2}{\sqrt{2}}\right)B, \\ \Omega_v &= B. \end{aligned} \quad (15)$$

From equation (15), we can obtain that the Ω 's quantities of fundamental forms are

$$\begin{aligned} E &= v^2\tau^2 + \frac{1}{2}(k_1 + k_2)^2, \\ F &= \frac{-1}{\sqrt{2}}(k_1 + k_2), \\ G &= 1, \\ \ell &= \left(\frac{k_2}{\tau}\right) \left[vk_1' - \frac{k_1(k_1 + k_2)}{\sqrt{2}} \right] - \left(\frac{k_1}{\tau}\right) \left[vk_2' - \frac{k_1(k_1 + k_2)}{\sqrt{2}} \right], \\ m &= 0, \\ n &= 0. \end{aligned} \tag{16}$$

Consequently, from the above data, we obtain K_Ω and H_Ω of the $\mu_1\mu_2$ type-2 Smarandache ruled surface given as follows:

$$\begin{aligned} K_\Omega &= 0, \\ H_\Omega &= \frac{k_1'k_2 - k_1k_2'}{2v\tau^3}. \end{aligned} \tag{17}$$

Also, we use (12) to get the normal curvature, the geodesic curvature, and the geodesic torsion that associate $\psi(\sigma)$ on Ω as the following:

$$\begin{aligned} \kappa_n &= 0, \\ \kappa_g &= -\frac{k_1^2}{\tau}, \\ \tau_g &= \frac{1}{\tau^3} \left[\tau^2 \left(\frac{k_1k_2}{\tau}\right)' - k_1k_2\tau' \right]. \end{aligned} \tag{18}$$

So, the proof ended. □

3.2. μ_1B Type-2 Smarandache Ruled Surface

Definition 7. For a regular curve $\psi = \psi(\sigma)$ in \mathbf{E}^3 related to the frame $\{\mu_1, \mu_2, B\}$, the μ_1B type-2 Smarandache ruled surface is given as

$$\Phi = \Phi(\sigma, v) = \frac{1}{\sqrt{2}}(\mu_1(\sigma) + B(\sigma)) + v\mu_2(\sigma). \tag{19}$$

Theorem 8. Let $\Phi = \Phi(\sigma, v)$ be the μ_1B type-2 Smarandache ruled surface in \mathbf{E}^3 defined by (19). Then, we have

- (1) If $k_1k_2 = 0$, then, Φ is a developable surface with the geodesic base curve
- (2) Φ is a minimal surface with the geodesic base curve if and only if the type-2 Bishop curvatures satisfy the following differential equation

$$\begin{aligned} &(k_1 + \sqrt{2}vk_2) \left[k_1' - k_1(k_1 + \sqrt{2}vk_2) \right] \\ &- k_1(\tau^2 + k_1' + \sqrt{2}vk_2') - 2k_1k_2' = 0. \end{aligned} \tag{20}$$

Proof. Considering the μ_1B type-2 Smarandache ruled surface given by (19), then, the velocity vectors of Φ are given as follows:

$$\begin{aligned} \Phi_\sigma &= \left(\frac{k_1}{\sqrt{2}}\right)\mu_1 + \left(\frac{k_2}{\sqrt{2}}\right)\mu_2 - \left(\frac{k_1 + \sqrt{2}vk_2}{\sqrt{2}}\right)B, \\ \Phi_v &= \mu_2. \end{aligned} \tag{21}$$

From equation (21), the Φ 's quantities of fundamental forms are

$$\begin{aligned} E &= \frac{1}{2} \left[\tau^2 + (k_1 + \sqrt{2}vk_2)^2 \right], \\ F &= \frac{k_2}{\sqrt{2}}, \\ G &= 1, \\ \ell &= \frac{(k_1 + \sqrt{2}vk_2) \left[k_1' - k_1(k_1 + \sqrt{2}vk_2) \right] - k_1 \left[\tau^2 + k_1' + \sqrt{2}vk_2' \right]}{\sqrt{2} \sqrt{k_1^2 + (k_1 + \sqrt{2}vk_2)^2}}, \\ m &= -\frac{k_1k_2}{\sqrt{k_1^2 + (k_1 + \sqrt{2}vk_2)^2}}, \\ n &= 0. \end{aligned} \tag{22}$$

Then, K_Φ and H_Φ of the μ_1B type-2 Smarandache ruled surface is given as follows:

$$\begin{aligned} K_\Phi &= -\frac{2k_1^2k_2^2}{\left[k_1^2 + (k_1 + \sqrt{2}vk_2)^2 \right] \left[\tau^2 - k_2^2 + (k_1 + \sqrt{2}vk_2)^2 \right]}, \\ H_\Phi &= \frac{(k_1 + \sqrt{2}vk_2) \left[k_1' - k_1(k_1 + \sqrt{2}vk_2) \right] - k_1 \left[\tau^2 + k_1' + \sqrt{2}vk_2' \right] + 2k_1k_2'}{\sqrt{2} \sqrt{k_1^2 + (k_1 + \sqrt{2}vk_2)^2}}. \end{aligned} \tag{23}$$

Furthermore, from (12), we have

$$\begin{aligned} \kappa_n &= \frac{(k'_1 - k_1^2)(k_1 + \sqrt{2}vk_2) - k_1(\tau^2 + k_1')}{\sqrt{2}\sqrt{k_1^2 + (k_1 + \sqrt{2}vk_2)^2}}, \\ \kappa_g &= 0, \\ \tau_g &= -\frac{k_1^2 k_2 (k_1 + \sqrt{2}vk_2)}{k_1^2 + (k_1 + \sqrt{2}vk_2)^2}, \end{aligned} \tag{24}$$

which replies to the above theorem.

3.3. $\mu_2 B$ Type-2 Smarandache Ruled Surface

Definition 9. For a regular curve $\psi = \psi(\sigma)$ in E^3 related to the frame $\{\mu_1, \mu_2, B\}$, the $\mu_2 B$ type-2 Smarandache ruled surface is given as

$$\Psi = \Psi(\sigma, v) = \frac{1}{\sqrt{2}}(\mu_2(\sigma) + B(\sigma)) + v\mu_1(\sigma). \tag{25}$$

Theorem 10. Let $\Psi = \Psi(\sigma, v)$ be the $\mu_2 B$ type-2 Smarandache ruled surface in E^3 defined by (25). Then, we have

- (1) If $k_1 k_2 = 0$, then, Ψ is a developable surface with the principal base curve
- (2) Ψ is a minimal surface if and only if the type-2 Bishop curvatures satisfy the following differential equation

$$k_2(\tau^2 + k_2' + \sqrt{2}vk_1') - 2k_1^2 k_2 - (k_2 + \sqrt{2}vk_1) \cdot [k_2' - k_2(k_2 + \sqrt{2}vk_1)] = 0. \tag{26}$$

Proof. Considering the $\mu_2 B$ type-2 Smarandache ruled surface given by (25), then, the velocity vectors of Ψ are given as follows:

$$\begin{aligned} \Psi_\sigma &= \left(\frac{k_1}{\sqrt{2}}\right)\mu_1 + \left(\frac{k_2}{\sqrt{2}}\right)\mu_2 - \left(\frac{k_2 + \sqrt{2}vk_2}{\sqrt{2}}\right)B, \\ \Psi_v &= \mu_1. \end{aligned} \tag{27}$$

From equation (27), the Ψ 's quantities of fundamental forms are

$$\begin{aligned} E &= \frac{1}{2} \left[\tau^2 + (k_2 + \sqrt{2}vk_1)^2 \right], \\ F &= \frac{k_1}{\sqrt{2}}, \\ G &= 1, \\ e &= \frac{k_2 \left[\tau^2 + k_2' + \sqrt{2}vk_1' \right] - (k_2 + \sqrt{2}vk_1) \left[k_2' - k_2(k_2 + \sqrt{2}vk_1) \right]}{\sqrt{2}\sqrt{k_2^2 + (k_2 + \sqrt{2}vk_1)^2}}, \\ m &= \frac{k_1 k_2}{\sqrt{k_2^2 + (k_2 + \sqrt{2}vk_1)^2}}, \\ n &= 0. \end{aligned} \tag{28}$$

The K_Ψ and H_Ψ of the $\mu_2 B$ type-2 Smarandache ruled surface given as follows:

$$\begin{aligned} K_\Psi &= -\frac{2k_1^2 k_2^2}{\left[k_2^2 + (k_2 + \sqrt{2}vk_1)^2 \right] \left[\tau^2 - k_1^2 + (k_2 + \sqrt{2}vk_1)^2 \right]}, \\ H_\Psi &= \frac{k_2(\tau^2 + k_2' + \sqrt{2}vk_1') - 2k_1^2 k_2 - (k_2 + \sqrt{2}vk_1) \left[k_2' - k_2(k_2 + \sqrt{2}vk_1) \right]}{\sqrt{2}\sqrt{k_2^2 + (k_2 + \sqrt{2}vk_1)^2}}. \end{aligned} \tag{29}$$

So, the proof ended. □

Also, from (12), we have

$$\begin{aligned} \kappa_n &= \frac{k_2(\tau^2 + k_2') - (k_2' - k_1^2)(k_2 + \sqrt{2}vk_1)}{\sqrt{2}\sqrt{k_2^2 + (k_2 + \sqrt{2}vk_1)^2}}, \\ \kappa_g &= \frac{k_1(k_2 + \sqrt{2}vk_1)}{\sqrt{k_2^2 + (k_2 + \sqrt{2}vk_1)^2}}, \\ \tau_g &= \frac{k_1^2 k_2 (k_2 + \sqrt{2}vk_1)}{k_2^2 + (k_2 + \sqrt{2}vk_1)^2}. \end{aligned} \tag{30}$$

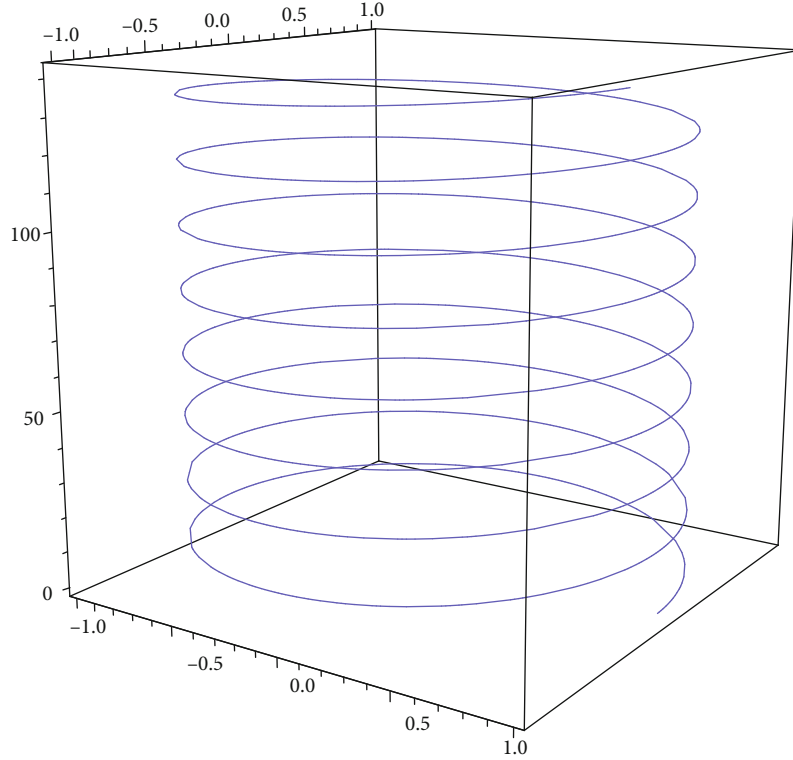


FIGURE 1: Curve $\psi = \psi(\sigma)$.

Then, equations (29) and (30) complete the proof.

3.4. Example. Let ψ be a circular helix parameterized as $\psi(\sigma) = (\cos(\sigma/3), \sin(\sigma/3), (2\sqrt{2}\sigma/3))$ (see Figure 1). Then, we have

$$\begin{aligned} T(\sigma) &= \left(-\frac{1}{3} \sin\left(\frac{\sigma}{3}\right), \frac{1}{3} \cos\left(\frac{\sigma}{3}\right), \frac{2\sqrt{2}}{3} \right), \\ N(\sigma) &= \left(-\cos\left(\frac{\sigma}{3}\right), -\sin\left(\frac{\sigma}{3}\right), 0 \right), \\ B(\sigma) &= \left(\frac{2\sqrt{2}}{3} \sin\left(\frac{\sigma}{3}\right), -\frac{2\sqrt{2}}{3} \cos\left(\frac{\sigma}{3}\right), \frac{1}{3} \right). \end{aligned} \tag{31}$$

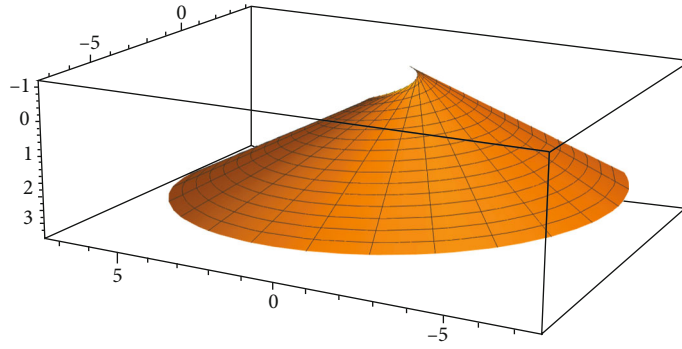
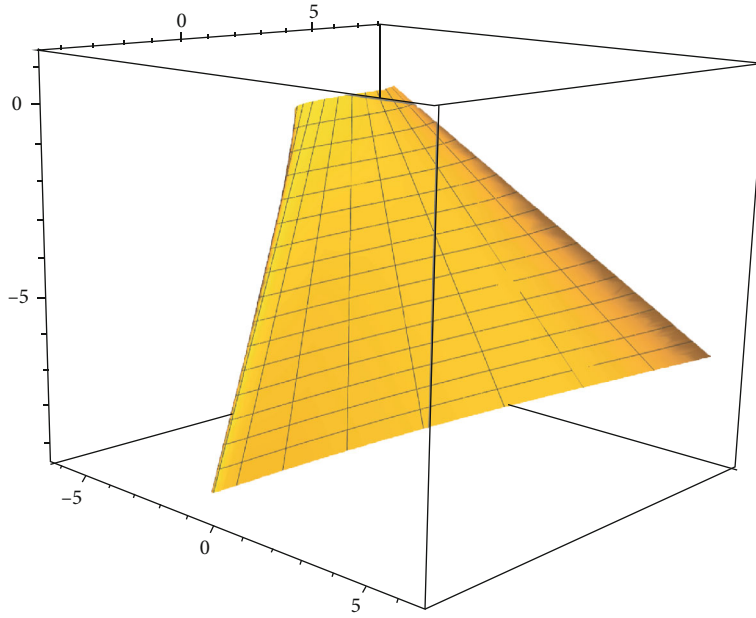
Then, $\tau = 2\sqrt{2}/9 \neq 0$ and $\theta(\sigma) = \int_0^\sigma (1/9) d\sigma = \sigma/9$. From (4), we get $\kappa_1(\sigma) = -(2\sqrt{2}/9) \cos(\sigma/9)$, $\kappa_2(\sigma) = -(2\sqrt{2}/9) \sin(\sigma/9)$. Also, we have

$$\begin{aligned} \mu_1(\sigma) &= \left(-\cos\left(\frac{\sigma}{9}\right) \cos\left(\frac{\sigma}{3}\right) - \frac{1}{3} \sin\left(\frac{\sigma}{9}\right) \right. \\ &\quad \cdot \sin\left(\frac{\sigma}{3}\right), \frac{1}{3} \cos\left(\frac{\sigma}{3}\right) \sin\left(\frac{\sigma}{9}\right) \\ &\quad \left. - \cos\left(\frac{\sigma}{9}\right) \sin\left(\frac{\sigma}{3}\right), \frac{2\sqrt{2}}{9} \sin\left(\frac{\sigma}{9}\right) \right), \end{aligned}$$

$$\begin{aligned} \mu_2(\sigma) &= \left(-\cos\left(\frac{\sigma}{3}\right) \sin\left(\frac{\sigma}{9}\right) + \frac{1}{3} \cos\left(\frac{\sigma}{9}\right) \sin\left(\frac{\sigma}{3}\right), \right. \\ &\quad \left. -\frac{1}{3} \cos\left(\frac{\sigma}{9}\right) \cos\left(\frac{\sigma}{3}\right) - \sin\left(\frac{\sigma}{9}\right) \sin\left(\frac{\sigma}{3}\right), \right. \\ &\quad \left. -\frac{2\sqrt{2}}{9} \cos\left(\frac{\sigma}{9}\right) \right). \end{aligned} \tag{32}$$

The $\mu_1\mu_2$ type-2 Smarandache ruled surface $\Omega(\sigma, v)$ (see Figure 2)

$$\begin{aligned} \Omega(\sigma, v) &= \left(\frac{1}{\sqrt{2}} \left\{ \frac{1}{3} \sin\left(\frac{\sigma}{3}\right) \left(\cos\left(\frac{\sigma}{9}\right) - \sin\left(\frac{\sigma}{9}\right) \right) \right. \right. \\ &\quad \left. \left. - \cos\left(\frac{\sigma}{3}\right) \left(\sin\left(\frac{\sigma}{9}\right) + \cos\left(\frac{\sigma}{9}\right) \right) \right\} \right. \\ &\quad \left. + \frac{2}{3} v \sin\left(\frac{\sigma}{3}\right), \frac{1}{\sqrt{2}} \left\{ \frac{1}{3} \cos\left(\frac{\sigma}{3}\right) \left(\sin\left(\frac{\sigma}{9}\right) \right. \right. \right. \\ &\quad \left. \left. - \cos\left(\frac{\sigma}{9}\right) \right) - \sin\left(\frac{\sigma}{3}\right) \left(\cos\left(\frac{\sigma}{9}\right) \right. \right. \right. \\ &\quad \left. \left. + \sin\left(\frac{\sigma}{9}\right) \right) \right\} - \frac{2}{3} v \cos\left(\frac{\sigma}{3}\right), \frac{2}{3} \right. \\ &\quad \left. \cdot \left(\sin\left(\frac{\sigma}{9}\right) - \cos\left(\frac{\sigma}{9}\right) + \frac{v}{3} \right) \right). \end{aligned} \tag{33}$$

FIGURE 2: $\mu_1\mu_2$ type-2 Smarandache ruled surface $\Omega(\sigma, \nu)$.FIGURE 3: $\mu_1 B$ type-2 Smarandache ruled surface $\Phi(\sigma, \nu)$.

The $\mu_1 B$ type-2 Smarandache ruled surface $\Phi(\sigma, \nu)$ is (see Figure 3)

$$\begin{aligned} \Phi(\sigma, \nu) = & \left(\frac{1}{3} \sin\left(\frac{\sigma}{3}\right) \left(\nu \cos\left(\frac{\sigma}{9}\right) - \frac{1}{\sqrt{2}} \sin\left(\frac{\sigma}{9}\right) \right) \right. \\ & - \cos\left(\frac{\sigma}{3}\right) \left(\nu \sin\left(\frac{\sigma}{9}\right) + \frac{1}{\sqrt{2}} \cos\left(\frac{\sigma}{9}\right) \right) \\ & + \frac{2}{3} \sin\left(\frac{\sigma}{3}\right), \frac{1}{3} \cos\left(\frac{\sigma}{3}\right) \left(\frac{1}{\sqrt{2}} \sin\left(\frac{\sigma}{3}\right) \right. \\ & \left. - \nu \cos\left(\frac{\sigma}{3}\right) \right) - \sin\left(\frac{\sigma}{3}\right) \left(\nu \sin\left(\frac{\sigma}{3}\right) \right. \\ & \left. + \frac{1}{\sqrt{2}} \cos\left(\frac{\sigma}{3}\right) \right) - \frac{2}{3} \cos\left(\frac{\sigma}{3}\right), \frac{2}{3} \\ & \left. \cdot \left(\sin\left(\frac{\sigma}{9}\right) - \sqrt{2} \nu \cos\left(\frac{\sigma}{9}\right) + \frac{1}{2\sqrt{2}} \right) \right). \end{aligned} \quad (34)$$

The $\mu_2 B$ type-2 Smarandache ruled surface $\Psi(\sigma, \nu)$ is (see Figure 4)

$$\begin{aligned} \Psi(\sigma, \nu) = & \left(\frac{1}{3} \sin\left(\frac{\sigma}{3}\right) \left(\frac{1}{\sqrt{2}} \cos\left(\frac{\sigma}{9}\right) - \nu \sin\left(\frac{\sigma}{9}\right) \right) \right. \\ & - \cos\left(\frac{\sigma}{3}\right) \left(\frac{1}{\sqrt{2}} \sin\left(\frac{\sigma}{9}\right) + \nu \cos\left(\frac{\sigma}{9}\right) \right) \\ & + \frac{2}{3} \sin\left(\frac{\sigma}{3}\right), \frac{1}{3} \cos\left(\frac{\sigma}{3}\right) \left(\nu \sin\left(\frac{\sigma}{3}\right) \right. \\ & \left. - \frac{1}{\sqrt{2}} \cos\left(\frac{\sigma}{3}\right) \right) - \sin\left(\frac{\sigma}{3}\right) \left(\frac{1}{\sqrt{2}} \sin\left(\frac{\sigma}{3}\right) \right. \\ & \left. + \nu \cos\left(\frac{\sigma}{3}\right) \right) - \frac{2}{3} \cos\left(\frac{\sigma}{3}\right), \frac{2}{3} \\ & \left. \cdot \left(\sqrt{2} \nu \sin\left(\frac{\sigma}{9}\right) - \cos\left(\frac{\sigma}{9}\right) + \frac{1}{2\sqrt{2}} \right) \right). \end{aligned} \quad (35)$$

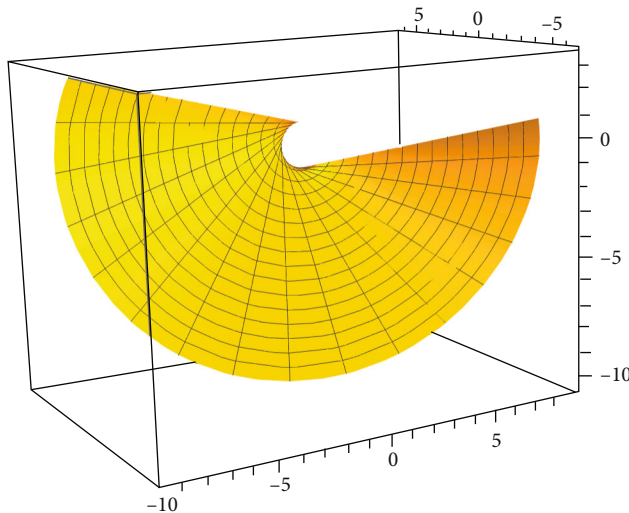


FIGURE 4: $\mu_2 B$ type-2 Smarandache ruled surface $\Psi(\sigma, \nu)$.

4. Conclusion

The study of ruled surfaces with different moving frames is one of the most interesting points of this paper. The researchers found that these surfaces could be developed in a minimal amount of time. In this work, we describe and study type-2 Smarandache ruled surfaces, which are a specific form of ruled surfaces. We create the essential and adequate circumstances for these surfaces to be developable in a minimal amount of time.

Data Availability

No data is used in this study.

Conflicts of Interest

The authors declare no competing interest.

Authors' Contributions

All authors have equal contributions and finalized the study.

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