The Darwin-Breit Interaction vs. the Aharonov-Bohm Potentials

E. Comay[1*]

[1]*Charactell Ltd., P.O.Box 39019, Tel-Aviv, 61390, Israel.*

Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

Original Research Article

ABSTRACT

The paper shows that the Darwin-Breit form of electromagnetic interaction disproves the Aharonov-Bohm assertion about the fundamental meaning of the electromagnetic potentials in quantum theory. A specific analysis of the Aharonov-Bohm electric and magnetic effects substantiates this result.

1 INTRODUCTION

About one hundred years ago C. G. Darwin proved that the electromagnetic interaction can be expanded in a power series of v/c. He has written a mechanical-like electromagnetic Lagrangian that accounts for terms up to v^n/c^n, where $n \leq 2$ (see [1], pp. 179-182; [2], pp. 593-595). This Lagrangian is independent of the electromagnetic 4-potential and fields. About ten years later, G. Breit published a quantum version of the interaction term of the Darwin Lagrangian. He used the Dirac velocity operator $\boldsymbol{\alpha}$ instead of the Darwin classical velocity $\boldsymbol{v}$ (see [3], pp. 170, 195). The publication of the Darwin Lagrangian and its quantum version, namely the Breit

Corresponding author: E-mail: elicomay@post.tau.ac.il;

interaction, in textbooks is followed by an adequate mathematical analysis. This issue provides a solid basis for the arguments that are presented below, which discuss implications of the Darwin-Breit work on the Aharonov-Bohm (AB) effects.

About 60 years ago, AB published two papers that describe the "significance of electromagnetic potentials in the quantum theory" [4, 5]. The second AB paper [5] indicates that pro and con reactions to their work have been made. V. Weisskopf has made a famous remark about the AB papers: "The first reaction to this work is that it is wrong; the second is that it is obvious" [6]. Other people have followed the AB idea, which is now regarded as an acceptable element of quantum theories. Article supporting the AB idea have been published until recently [7].

However, the mathematical coherence of a physical idea is superior to public opinion. Here the Darwin-Breit formulation of interacting charged particles shows that the electromagnetic 4-potential can be removed. In contrast, AB argue that the 4-potential is a significant part of quantum theories of a charged particle. This apparent contradiction deserves a closer analysis. Furthermore, any new examination of a theoretical idea can only improve the understanding of the relevant topic. The present work undertakes this assignment and provides new arguments concerning the Darwin-Breit interaction and the AB assertion.

The paper uses standard notation. The second section proves that the Darwin-Breit interaction refutes the main AB idea. The third section proves that the (never detected) electric AB effect violates energy conservation. The fourth section proves an erroneous element in the AB interpretation of their magnetic effect. The Tonomura experiment [8] that is related to the magnetic AB effect is analyzed. The last section contains concluding remarks.

2 SIGNIFICANCE OF THE DARWIN LAGRANGIAN AND THE BREIT INTER-ACTION

As stated in the introduction section, the Darwin Lagrangian belongs to classical electrodynamics. It shows that a velocity-dependent power series expansion of the charged two-body interaction yields a mechanical-like instantaneous two-body Lagrangian. The interaction part of this Lagrangian is

$$L_{Darwin} = -\sum_j \sum_{i>j} \frac{e_j e_i}{R_{ij}} + \sum_j \sum_{i>j} \frac{e_j e_i}{2R_{ij}} [\boldsymbol{v}_j \cdot \boldsymbol{v}_i + (\boldsymbol{v}_j \cdot \boldsymbol{n}_{ij})(\boldsymbol{v}_i \cdot \boldsymbol{n}_{ij})], \tag{1}$$

where i, j run on the system's charged particles. This Lagrangian is obtained from a power series expansion of the classical Lagrangian in terms of v/c, and it accounts for quantities up to v^n/c^n, where $n \leq 2$. It is clearly independent of electromagnetic potentials and fields.

The Breit interaction is the quantum version of the interaction term of the Darwin Lagrangian. It uses the Dirac velocity operator $\boldsymbol{\alpha}$ instead of the Darwin velocity $\boldsymbol{v}$ (see [3], pp. 170, 195). As a power series expansion up to v^2/c^2, the Darwin-Breit interaction certainly applies to low-velocity systems of nonrelativistic theories. It turns out that the AB analysis considers this kind of system. For example, in [4] they state; "In the nonrelativistic limit (and we shall assume this almost everywhere in the following discussions)...". In the second AB article [5], they speak on adiabatic changes of the potential, which mean a very slow process. Furthermore, textbooks that mention the AB effect discuss the nonrelativistic version of the Heisenberg-Schroedinger quantum theory, and most, if not all, textbooks on relativistic quantum theory do not mention the effect. Hence, the community assigns the effect to the non-relativistic domain.

Conclusion: The Darwin-Breit interaction removes not only the 4-potentials but also the electromagne-tic fields. One result of the applicability of this matter to non-relativistic quantum mechanics means that the potentials are not a crucial element of non-relativistic quantum theories. It follows that the entire AB assertion collapses because a fundamental element of a given theory cannot be removed from any coherent form of this theory.

This result can be considered as a conclusive theoretical argument against the AB assertion. However, one may argue that something may be wrong with the Darwin-Breit expression and not with the AB theory. Therefore, an analysis of specific AB cases is required. This analysis shows specific theoretical errors in the AB work. In so doing, it substantiates the outcome of the Darwin-Breit theory and illuminates relevant issues. Hence, it helps readers to acquire a better understanding of the problem. The rest of this work is dedicated to some of these topics.

3 A COUNTEREXAMPLE

The electric AB effect has never been confirmed experimentally [9]. This section describes a counter-example to the electric AB effect. This example supports the theoretical arguments of the previous section and explains the failure to detect the electric AB effect.

Fig. 1 demonstrates the electric AB effect [4]. (Due to limitation of the page's width, the x-axis of the figure is contracted significantly.) An electronic beam moves from left to right, and at a point outside the figure, it is chopped into well-separated portions. Later, at another point (still unseen in the figure) a beam splitter splits it into two sub-beams that continue to move rightwardly. A black circle describes the position of an electron of each sub-beam at a specific instant, and the arrow shows the direction of its motion. The apparatus comprises an upper pair and a lower pair of large thin plates made of insulating material. Each plate is covered uniformly with positive or negative charge of the same density. Hence, each plate is an inert object because its own state does not change, and all its elements can only move as a rigid body. The state of the apparatus changes during the experiment and the figure shows the position of its plates at three time-intervals. At $t, t < t_1$ the distance between the two plates of each pair is infinitesimal. Considering the plates' opposite charge, one finds that at this period the plates' electric potential and their fields cancel each other at the electron's position.
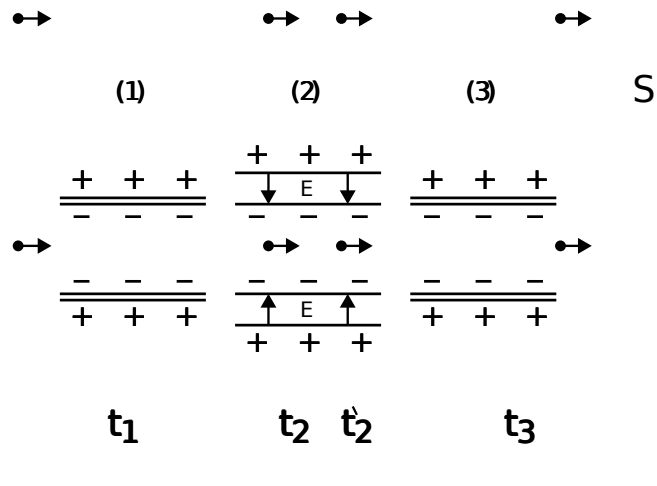

Fig. 1. The electric AB effect (see text)

Let us examine an electron of the lower sub-beam. At t_2 the electron is well inside the plates. Around this time an engine of the apparatus drives outwards the positively charged plates and they reach the position shown on the middle part of the figure. The electron is far from the plates' edge and it continues its motion in a field-free region. However, an electric field $\mathbf{E}$ arises, and the electric potential is negative at the electron's location. At t_2' the electron is still well inside the plates, and the positively charged plates return to their original position. At t_3 the electron exits the plates' region and continues its motion towards the interference screen S.

These arguments show that the electron of the lower sub-beam travels at a field-free region. Its potential at different instants was as follows: For t, $t < t_2$ it moved in a null potential; for t, $t_2 < t < t_2'$ it moved in a negative potential; for t, $t > t_2'$ it moved in a null potential. During this time, an electron of the upper sub-beam moved in a null potential, field-free region. Electrons of the two sub-beams interfere on the screen S.

AB argue that the potential affects the phase of an electron of the lower sub-beam. As a result, the interference pattern changes. In their opinion, this outcome proves the physical meaning of the electric potential.

The following argument disproves the AB assertion. Let us examine a modification of the previous experiment (see fig. 2). Until t_2', the motion is the same as that of fig. 1. In the second experiment at t, $t > t_2'$ the state of the plates remains unchanged until the electron exits the plates' region. Due to the negative potential between the two pairs of plates, the negatively charged electron of the lower sub-beam increases its kinetic energy. It enters a device D which removes the additional kinetic energy, and the electron restores its original kinetic energy. The screen S of the previous experiment is removed. For each sub-beam, appropriate magnetic fields change the direction of the electron's motion, unify the two sub-beams, and brings them back to the left-hand side of the apparatus, where the unified beam enters the beam chopper. During the latter period, the positively charged plates return to their original position.

If AB are right then the net outcome of this process is a gain of energy by the device D and a violation of the energy conservation law. The foregoing discussion not only disproves the theoretical basis of the electric AB effect, but also shows where the original AB flaw lies.

AB claim in their second paper that they extend the argument of the first paper and "include the sources of potentials quantum-mechanically, and we show that when this is done, the same results are obtained as those of our first paper in which the potential was taken to be a specified function of space and time."
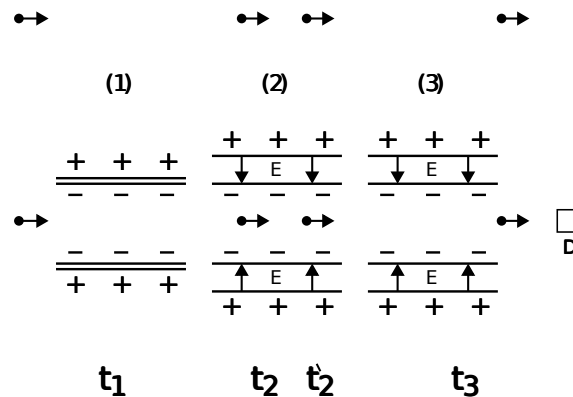

Fig. 2. A refutation of the electric AB effect (see text)

This is not true. Let the single-particle state of an electron of the split sub-beam be

$$\psi = a\psi_o(x,t) + b\psi_i(x,t), \tag{2}$$

where x, t denote the space-time coordinates, the subscripts o, i denote an electron that is outside/inside the plates' region, respectively, and a, b are numerical constants. Let Ψ denote the quantum state of the apparatus. Evidently, the net effect of $\psi_o(x)$ on the plate's motion vanishes. In contrast, $\psi_i(x)$ resists the outward motion of the two plates. Hence, if the electron is inside the plates' region then at t, $t_2 < t < t_2'$ the plates' self-energy of the state $\psi_i(x)$ is a little bit smaller than that of the case described by $\psi_o(x)$. The combined quantum mechanical state is

$$\Phi = a\psi_o(x,t)\Psi_o(y_j,t) + b\psi_i(x,t)\Psi_i(y_j,t), \tag{3}$$

where y_j denote the coordinates of elements of the apparatus.

Eq. (3) agrees with eq. (19) of the second AB paper [5]. Below (19) AB write: "... because the parts of the source are so heavy, only a single such product is actually needed." The experiment described above demonstrates the incorrectness of this AB statement. Evidently, at t, $t_2 < t < t_2'$ the self-energy of the apparatus' function $\Psi_i(y_j)$ is smaller than that of $\Psi_o(y_j)$. Energy conservation of electrodynamics proves that this is precisely the additional energy deposited by the electron at the device D.

AB ignore this difference and state: (see [5], p. 1519) "We have thus accomplished our objective of showing that when the source of potential is taken into account quantum-mechanically, we obtain the same result as that given in our first paper, where the potential was assumed to be a specified function of space and time."

The counterexample described herein denies this AB assertion and proves that their electric effect violates energy conservation. And indeed, as stated above [9], the electric AB effect has never been detected experimentally. The discussion of this section provides a theoretical explanation for this failure.

4 THE MAGNETIC AB EFFECT

The magnetic AB effect has already been discussed [10]. This paper proves that contrary to a fundamental AB claim, topological features of the force-free region cannot be regarded as a necessary and sufficient condition for a nonzero phase shift, and that topology takes no profound physical role. On the other hand, the paper [10] proves the crucial role of an inert object as the source of the magnetic field. And indeed, the Tonomura experiment [8], which demonstrated the magnetic AB effect, uses an inert magnetic object. Furthermore, a classical source of the fields is not inert, and it will provide no effect.

Fig. 2 describes the Tonomura experiment. $\mathbf{M}$ is a circular single domain magnet that is made of a ferromagnetic material, and $\mathbf{B}$ denotes its magnetic field. An electronic beam moves upwards. $\mathbf{e}$ denotes an electron of the beam that passes through the inner part of the magnet, and $\mathbf{Be}$ denotes the magnetic field of this electron. $\mathbf{e}'$ denotes another trajectory of the beam, that passes at the outer region of the magnetic single domain.
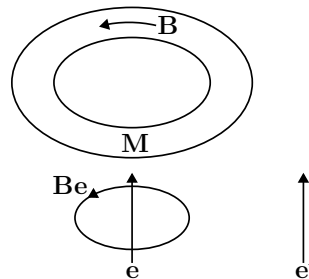

Fig. 3. The Tonomura experiment (see text)

The dual nature of the 2-body electromagnetic interaction is the primary point that denies the AB topological field-free region. Here are two completely equivalent calculations: One may consider the field-free region of an electron of the beam and argue that the interaction is $e\,\boldsymbol{v}\cdot\boldsymbol{A}$. The time-integral of this quantity yields the additional phase. This form apparently proves the AB claim. In contrast, one may argue that the interaction is the magnetic elements of the single domain that interact with the magnetic field of the moving electron $-\boldsymbol{\mu}\cdot\boldsymbol{B_e}$ (see [2] p. 186). The time-integral of this quantity yields the additional phase. In the second picture, the field is associated with the moving electron, and there is no field-free region. Obviously, a fundamental physical property should not disappear if an alternative legitimate calculation is carried out. Hence, the entire AB concept collapses. The legitimacy of the second picture proves that, contrary to the primary AB claim, the topological field-free region is not an essential element of the magnetic AB effect. On the other hand, as proved in [10], AB have missed the crucial role of an inert source for the magnetic effect.

5 CONCLUDING REMARKS

Aharonov and Bohm argue that the electromagnetic potentials are indispensable elements of the non-relativistic quantum theory. On the other hand, the Darwin-Breit interaction proves that the nonrelativistic quantum theory can be consistently written without potentials. Hence, the AB assertion collapses.

An analysis of the electric AB effect (which has never been detected experimentally) proves that this suggested effect violates energy conservation. This result supports the Darwin-Breit approach. Furthermore, an analysis of the magnetic AB effect and the Tonomura experiment proves erroneous elements of the AB analysis: A topological multiply connected field-free region is not an essential requirement; the inert source of the magnetic field is required for a phase shift of the sub-beam. Hence, contrary to the AB assertion, the magnetic AB effect is an ordinary quantum effect, because an inert magnetic source is a spin-dependent quantum system [10].

Some results of this work have been obtained earlier [10]. The analysis of [11] also arrived at some results of this work: "It removes, however, conceptual claims associated with the AB effect regarding nonlocality and the meaning of potentials. The AB effect does not prove that the evolution of a composite system of charged particles cannot be described completely by fields at locations of all particles. The potentials might be just a useful auxiliary mathematical tool after all."

Relativistic problems with the 4-potential of a photon are already pointed out in a textbook: "the fact that A^0 vanishes in all Lorentz frames shows vividly that A^μ cannot be a four-vector" (see [12], p. 251). Despite this problematic state, it is shown in [13] that a relativistically compatible transformation of the 4-potential of radiation fields can be performed.

General problems of the electromagnetic 4-potential are discussed in [14], where an analysis of well-established experimental data proves that radiation fields and bound fields are inherently different physical objects. Hence, a distinction between bound fields and radiation field is an imperative element of a coherent electromagnetic theory. The Darwin-Breit treatment of bound electromagnetic fields is compatible with this conclusion.

The novelty of this work follows this distinction. Referring to the electromagnetic 4-potential, it goes one step further: Relying on the Darwin and Breit Lagrangian, it removes not only the 4-potentials but also the bound electromagnetic fields that are involved in the AB process! The removal of the potential and the bound electromagnetic fields means that the Darwin-Breit Lagrangian is *nonlocal.* However, unlike the long range of the EPR effect (see [15], pp. 375-381), the fast decrease of the $1/r_{12}$ factor of (1) indicates the short range of this nonlocality. Moreover, the Darwin Lagrangian belongs to classical physics, whereas the EPR is a quantum issue.

Conclusions: the electric AB effect is wrong because it violates energy conservation; AB have provided an erroneous explanation to the magnetic AB effect because they failed to recognize the crucial role of the inert

source of the potential; these results prove the validity of the Darwin-Breit interaction and the contradictions of the AB theory are explained.

DISCLAIMER

Author has declared that no competing interests exist. The products used for this research are commonly and predominantly use products in our area of research and country. There is absolutely no conflict of interest between the authors and producers of the products because we do not intend to use these products as an avenue for any litigation but for the advancement of knowledge. Also, the research was not funded by the producing company rather it was funded by personal efforts of the authors.

COMPETING INTERESTS

Author has declared that no competing interests exist.

REFERENCES

[1] Landau LD, Lifshitz EM. The classical theory of fields. Elsevier, Amsterdam; 2005.

[2] Jackson JD. Classical electrodynamics. 2nd edition, John Wiley, New York; 1975.

[3] Bethe HA, Salpeter EE. Quantum mechanics of one- and two-electron atoms. Springer, Berlin; 1957.

[4] Aharonov Y, Bohm D. Significance of electromagnetic potentials in the quantum theory. Phys. Rev. 1959;115:485.

[5] Aharonov Y, Bohm D. Phys. Rev. 1961;123:1511.

[6] Available: https://arxiv.org/pdf/1304.4736.pdf

[7] Popescu S. Dynamical quantum non-locality. Nature Physics. 2010;6:151-153.

[8] Tonomura A, Osakabe N, Matsuda T, Kawasaki T, Endo J, Yano S, Yamada H. Phys. Rev. Lett. 1986;56:792.

[9] Available:https://en.wikipedia.org/wiki/-Aharonov

[10] Comay E. Phys. Rev. A. 2000;62:042102-1.

[11] Vaidman L. Role of potentials in the Aharonov-Bohm effect. Phys. Rev. A. 2012;86:040101.

[12] Weinberg S. The quantum theory of fields. Cambridge University Press, Cambridge. 1995:1.

[13] Comay E. Acta physica polonica A. 2018;133:1294.
Available: http://przyrbwn.icm.edu.pl/APP-PDF/133/app133z5p26.pdf

[14] Comay E. The rise and fall of the electromagnetic 4-potential. OALib. 2018;5:1-18.
Available:https://www.scirp.org/journal/-PaperInformation.aspx?PaperID=88593

[15] Griffiths DJ. Introduction to quantum mechanics. Printice Hall, NJ; 1995.