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The Number of Four Corner Magic Squares of Order Six

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

In this paper we introduce and study special types of magic squares of order six. We present the property preserving transformations. We list some enumerations of these squares, which are computed using codes based on parallel computing.

Keywords: Magic squares; four corner property; parallel computing.

1. INTRODUCTION

In this paper we consider the old famous problem of magic squares. A semi magic square is a square matrix, where the sum of all entries in each column or row yields the same number. Some authors call it magic square. This number is called the magic constant. We call a semi magic square a magic square if both main diagonals sum up to the magic constant. A natural magic square of order *n* is a matrix of size $n \times n$ such that its entries consist of all integers from one to n^2. The magic constant in this case is $\frac{n(n^2+1)}{2}$. A symmetric magic square is a natural magic square of order *n* such that the sum of both elements of each pair of dual (opposite entries) equals n^2+1. For example, we see in Table 1 a natural symmetric magic square.

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Table 1. A natural symmetric magic square [3]

15	14	1	18	17
19	16	3	21	6
2	22	13	4	24
20	5	23	10	7
9	8	25	12	11

We note for example that we have the duals (15,11), (1,25), (21,5) etc.

It is well known that we have only eight 3x3 magic squares (with sum in all directions 15). All these squares have the number 5 as a middle entry and all these squares can be formed using the following transformations: rotations with angles $90^{\circ}, 180^{\circ}, 270^{\circ}$ and reflections about the middle column, middle row and both diagonals of the square

8	1	6
3	5	7
4	9	2

The combinations, which appear in the columns, rows and both diagonals of this square, are the only distinct three element combinations of the numbers from 1 to 9 with sum 15.

A pandiagonal magic square is a magic square such that the sum of all entries in all broken diagonals equals the magic constant. For example, we note in Table 2 that the sum of the entries 34, 36, 7, 44, 10, 2, 42 is 175, which is the magic constant. These entries represent the first right broken diagonal.

Table 2. A natural pandiagonal and symmetric magic square of order seven [3]

39	34	21	35	8	37	1
9	12	36	24	19	48	27
30	17	46	7	32	3	40
6	28	25	22	44	5	45
18	43	4	33	20	10	47
31	26	14	38	41	23	2
42	15	29	16	11	49	13

In the seventeenth century F. Bessy was the first person to state that the number of the 4x4 magic squares is 880, where he considered a magic square with all its rotations and reflections one square. Hire listed later these squares in tables in the year 1693. Today we can use the computer to check that there are

880*8 = 7040

magic squares of order 4. At the beginning of the twentieth century these squares were classified theoretically into twelve classes. One of these classes is the class of pandiagonal magic squares consisting of 48 squares. It was proven that they are generated by three basic squares (cf. [1]). In [2,3] we find a study of Frankiln's squares, which are magic squares of order 8. In [4,5] we find studies of some special magic squares like most-perfect pandiagonal magic squares or compound magic squares.

In 1973 Schoeppel found the number of all natural magic squares of order five. He computed it using an elementary computer. It is

64 826 306 * 32=2 202 441 792

where we multiply by 32 due to the existence of a property preserving transformations. According to [6] there exists

736 347 893 760

natural nested magic squares of order six.

It is well-known that there are pandiagonal magic squares and symmetric squares of order five. It was proven that the pandiagonal magic squares are generated through 144 basic squares. Hence, there are

144 * 200=28 800

natural pandiagonal squares of order 5. In order to understand this we consider the basic 5 by 5 pandiagonal square

A	B	C	D	E
F	G	H	I	J
K	L	M	N	O
P	Q	R	S	T
U	V	W	Y	Z

It is easy to see that by swapping all rows and columns of a pandiagonal square it remains pandiagonal. By swapping rows and columns we can ensure that: A is to be less than the other entries giving 25 possibilities, B < E so the B <--> E transformation arises giving 2 more possibilities, F < U so the F <--> U transformation arises giving 2 more possibilities and B < F so the B <--> F transformation arises giving 2 more possibilities giving 25*2*2*2 = 200 possibilities.

But, there are neither pandiagonal magic squares nor symmetric squares of order six. The proof can be found in [7]. The number of natural magic squares of order six is actually unknown up to day. Trump obtained using empirical methods (Monte Carlo Method) the following interval estimation for this number

(1.7712 e19, 1.7796 e19)

with a probability of 99%. We give here the number of a subset of such squares. We define here classes of magic squares of order six, which satisfy some of the conditions for both types.

The number of complete magic squares of order four is 48, and the number of complete magic squares of order eight (cf. [8]) is 368 640.

A pandiagonal and symmetric magic square is called super magic. According to [9] the number of super magic squares of order five is sixteen and number of super magic squares of order seven is

20 190 684.

The weakest property of a square is being semi magic. In the twentieth century many researchers worked on this topic. According to [12,9] the number of semi magic squares of order four is 68 688, and the number of semi magic squares of order five (computed by using the computer) is

579 043 051 200.

A basic semi magic square is one which represents 2(n!)^2 semi magic squares. There is only one basic natural semi magic square of order 3 of the form

a	b	c
d	e	f
g	h	l

where a is less than the other 8 entries, b < c, d < g and b < d. Swapping the 3 columns represents 3! possibilities and swapping the 3 rows represents 3! and b < d represents transposition. Thus, a basic 3 by 3 semi magic square represents 2(3!)^2 = 72 semi magic squares. A basic semi magic square 5 by 5 has the structure

a	b	c	d	e
f	g	h	i	j
k	l	m	n	o
p	q	r	s	t
u	v	w	y	z

where a is less than the other 24 entries, b < c < d < e, f < k < p < u and b < f. The number of basic natural semi magic squares 4 by 4 is 477 giving a total number of

477*2*(4!)^2 = 549 504 = 68 688 * 8

unique natural semi magic squares 4 by 4 (see [10,11]). Due to the equation

579 043 051 200 * 8 = 160 845 292 * 2 * (120)^2
We conclude that: the number of basic natural semi magic squares 5 by 5 is

160 845 292.

2 FOUR CORNER MAGIC SQUARE

This concept was introduced by Al-Ashhab for the first time in [10]. Al-Ashhab studied this type there in some simple cases. In [10] Al-Ashhab considered the type called nested four corner magic square with a pandiagonal magic square, where the inside 4 by 4 square was pandiagonal. We extend here the study of this type. In this section we use some similar concepts and ideas to those presented in [11,12]. But, we obtain supplementary results.

We focus in this paper on the following kind of magic squares: magic squares of order six with magic constant 3s such that

$$a_{ij} + a_{(i+3)(j+3)} + a_{i(j+3)} + a_{(i+3)j} = 2s$$

holds for each i=1,2,3 and j=1,2,3 and

$$a_{33} + a_{44} + a_{34} + a_{43} = 2s.$$

We call such squares four corner magic square of order 6. The entries of a four corner magic square of order 6 satisfy

$$a_{13} + a_{22} + a_{31} + a_{46} + a_{55} + a_{64} = 3s,\ a_{14} + a_{25} + a_{36} + a_{41} + a_{52} + a_{63} = 3s$$

These two conditions represent the sum of the entries of two broken diagonals. If the magic square is pandiagonal, then we have to consider all broken diagonals. To see the validity of the first equation we know from the definition that

$$a_{61} + a_{31} + a_{34} + a_{64} = 2s,$$
$$a_{22} + a_{25} + a_{52} + a_{55} = 2s,$$
$$a_{13} + a_{16} + a_{43} + a_{46} = 2s$$

holds. Adding up these equations and subtracting from them the following equation

$$a_{16} + a_{25} + a_{34} + a_{43} + a_{52} + a_{61} = 3s$$

yields the desired equation To see the validity of the second equation we know from the definition that

$$a_{33}+a_{66}+a_{36}+a_{63}=2s,\ a_{22}+a_{55}+a_{25}+a_{52}=2s,\ a_{11}+a_{44}+a_{14}+a_{41}=2s$$

holds. Adding up these equations and subtracting from them the following equation

$$a_{11}+a_{22}+a_{33}+a_{44}+a_{55}+a_{66}=3s$$

yields the desired equation. A four corner magic square of order 6 can be written as illustrated in Table 3.

Table 3. A symbolic four corner magic squares

y	f	g	t	M	G
z	h	n	j	q	N
w	E	e	a	m	D
A	k	2s –a–b–e	b	H	R
2s–j–o–z	p	d	o	2s–p–q–h	T
B	F	I	J	L	Y

Where

A= 2s – b – t – y,
B = b + j + o + t – s – w,
D = d + g + n + y – a – p – q,
E = 3s – a – e – m – w – D,
F = 3s– f – h – k – p – E,
G = 2s + e + w – (j + o + p + q + t),
H = e + g + s + w + y – j – k – o – p – q,
I = a + b + s – d – g – n,
J = 3s – a – b – j – o – t,
M = 3s – f – g – t – y – G,
N = 3s – h – j – n – q – z,
L = f + h + k + p – m – s,
R = a + b + j + o + p + q + t – g – 2s – w,
T = h + j + q+ z – d – s,
Y = p + q + s – b – e – y.

We see that it has seventeen independent variables, which are represented by the small letters. In the code these variables will be assigned to loops. We give in Table 4 an example of this type.

Table 4. A natural four corner magic squares [3]

6	23	11	13	33	25
19	28	36	3	7	18
2	29	1	17	27	35
21	8	22	34	10	16
32	9	15	20	30	5
31	14	26	24	4	12

2.1 Four Corner Magic Square with Negative Center or Positive Center

We introduce now the main concept in our work. We call a four corner magic squares such that

$$a_{33} * a_{44} - a_{34} * a_{43} < 0\ (> 0)$$

a four corner magic square of order six with negative (positive) center. This means that the 2 by 2 square in the center has negative (positive) determinant.

The number of all different possible values for a, b and e by computing the number of four corner magic squares is 3429. Hence, there are 3429 possible centers of the natural four corner magic squares. The number of squares with positive center is 232. Hence, there are 3197 possible centers of the negative four corner magic squares. These squares include the squares with symmetric centers (cf. [13]) and semi symmetric centers (cf. [11]). The number of centers for both types is 459. We are here interested in the squares, whose center is neither symmetric center nor semi symmetric center.

2.2 Property Preserving Transformations

There are seven classical transformations, which take a magic square into another magic square. These transformations also preserve the property "four corner magic". Now, a four corner magic squares can be transformed as follows into another one of the same kind: we make these interchanges simultaneously: interchange a_{12} (res. a_{62}) with a_{15} (res. a_{65}), interchange a_{21} (res. a_{26}) with a_{51} (res. a_{56}), interchange a_{22} (res. a_{55}) with a_{25} (res. a_{52}), interchange a_{23} (res. a_{24}) with a_{53} (res. a_{54}), interchange a_{32} (res. a_{42}) with a_{35} (res. a_{45}).

It is obvious that the center remains unchanged by this transformation. This means that a square with negative (positive) center will be transformed into another one of the same kind. We can use this transformation to reduce the number of computed natural magic squares. In order to eliminate the effect of the previous transformations we compute all natural four corner magic squares for which the following conditions hold:

$$a < 2s - a - b - e \,,\; a < e < b \,,\; p < q.$$

This means that we compute first the number of all natural squares satisfying these conditions. We multiply then this number by sixteen in order to get the number of squares.

2.3 Number of Squares

We used computers to count several types of magic squares. The algorithm is constructed in such a way that we take specific values at the beginning. In the case of four corner magic squares with negative center we fix by each run of the code two specific values for a, b and e, which satisfy the following conditions

$$a < e < b, \quad 8 < a < 18 \,,\; 2s - a - b - e > a,$$
$$a^*(2s - a - b - e) > b^*e$$

We exclude the squares with symmetric centers and semi symmetric centers in the next list. We list the number for all squares with respect to different values of a, b and e in the following tables: In Table (5-12) we list the squares corresponding to a = 9 (res. 10, …, 16).

Table 5. A list of the number of four corner magic squares with a = 9

b	e	Number	b	e	Number	b	e	Number
18	16	119393819	20	14	127374904	21	15	117413794
18	17	123352016	20	15	118762759	21	17	112788017
19	15	123662397	20	16	117910941	21	18	114191892
19	16	121759209	20	18	115356869	21	19	116805362
19	17	131768730	20	19	118351247	21	20	122373478
*	*	*	21	14	128835723	*	*	*

b	e	number	b	e	number	b	e	number
22	13	123752144	22	19	119115855	23	16	121807251
22	14	118433887	22	20	126616800	23	17	114746404
22	16	120650285	23	12	130932808	23	18	124584159
22	17	110774224	23	13	123262436	23	20	123651341
22	18	121100417	23	15	118203044	23	22	128145759

b=24

e	number	e	number	e	number	e	number
12	129257111	16	122029111	19	122452951	21	131340801
14	124319695	18	127377699	20	133211322	22	125439935
15	123377288	*	*	*	*	23	133319772

b=25

e	number	e	number	e	number	e	number	e	number
11	136852263	14	126558532	17	120247808	21	125793496	23	127789752
13	135409443	16	122798548	18	126016302	22	128757367	24	128695654
*	*	*	*	19	125211739	*	*	*	*

b=26

e	number	e	number	e	number	e	number	e	number
12	145534400	16	121918308	18	120791061	21	121782040	23	127833984
14	124919657	17	114958474	19	126550038	22	122293967	24	135534100
15	125273859	*	*	20	127670716	*	*	25	111520176

b=27

e	number	e	number	e	number	e	number	e	number
12	154387879	15	123710879	18	121544986	21	121486844	24	135585192
13	136035450	16	124687761	20	121296344	22	129742495	25	143788479
14	129947146	17	120752368	*	*	23	131519344	26	159581692

b=29

e	number	e	number	e	number	e	number
10	144128610	14	123966979	19	137394269	23	134995625
11	140116797	15	123803137	20	125212272	24	137128136
12	131864734	16	124049141	21	127862965	25	148226915
13	127857178	17	130357199	22	128552360	26	153398725

b=30

e	number	e	number	e	number	e	number
10	132552280	14	134866271	18	138084542	22	136026049
11	141024096	15	123482320	19	129121529	23	140739996
12	133847781	16	123972711	20	134339923	24	152674866
13	129741420	17	133650736	21	140106307	25	143150881

b	e	number	b	e	number	b	e	number
31	10	134735359	31	20	137858144	32	15	127901959
31	11	135960080	31	21	138696770	32	16	131369647
31	12	138087286	31	22	144550112	32	17	127993820
31	13	131554136	31	23	145281709	32	18	132006986
31	14	133185847	31	24	145487575	32	19	133206738
31	15	124467999	32	10	141540613	32	20	142422589
31	16	126377835	32	11	134646698	32	21	140125945
31	18	128300599	32	12	133884464	32	22	142915847
31	19	132196737	32	13	134300418	32	23	146679031
*	*	*	32	14	126405309	*	*	*

b	e	number	b	e	number	b	e	number
33	10	135686259	33	19	139405882	34	14	142039752
33	11	139434980	33	20	145101120	34	15	137390717
33	12	135693912	33	21	144462604	34	16	139595231
33	13	130829365	33	22	146603808	34	17	140537565
33	14	135218481	34	10	139581464	34	18	137997817
33	15	129499607	34	11	134512950	34	19	143463063
33	17	131322834	34	12	132699010	34	20	144187692
33	18	136789515	34	13	134159098	34	21	149214700

b	e	number	b	e	number	b	e	number
35	10	139151292	35	18	140866305	36	13	136582416
35	11	139164148	35	19	150772851	36	14	139031423
35	12	136711784	35	20	146484868	36	15	138421213
35	13	138664151	36	10	143963285	36	16	140420584
35	14	144148808	36	11	139067086	36	17	143252037
35	16	145865748	36	12	140422132	36	18	144833202
35	17	141822477	*	*	*	36	19	156154873

The number of centers is 183. The total number of the squares with a = 9 is 24 126 017 814

Table 6. A list of the number of four corner magic squares with a = 10

b	e	number	b	e	number	b	e	number
18	17	119554973	21	17	114569570	22	19	120465231
19	16	116099403	21	18	113047908	23	13	124319695
19	17	115356869	21	19	121594118	23	15	116963054
20	15	120107671	21	20	124594623	23	16	118858318
20	16	112788017	22	14	118203044	23	17	119898468
20	18	110552567	22	16	118417701	23	19	124901563
20	19	116497767	22	17	117601428	23	20	124448115
21	14	119808771	22	18	114368512	23	21	130357919
21	15	120650285	*	*	*	23	22	127579161

b	e	number	b	e	number	b	e	number
24	12	135409443	24	22	122094637	25	18	117984880
24	14	124569278	24	23	129444584	25	19	127046582
24	15	125597483	25	13	134159325	25	20	127101891
24	17	123924646	25	15	128669997	25	21	122693147
24	18	117604982	25	16	123688776	25	22	124160058
24	19	127142446	25	17	121250300	25	23	130931690
24	21	132115832	*	*	*	25	24	132367142

b=26

e	number	e	number	e	number	e	number
13	132814270	16	120140520	20	120144630	23	122786061
14	125824360	17	118268595	21	120616795	24	136590042
15	119430806	18	118534435	22	126132561	25	143210116

b=28

e	number	e	number	e	number	e	number
11	154387879	15	124687761	19	121296344	22	131519344
12	136035450	16	120752368	20	123192197	23	135585192
13	129947146	17	121544986	21	129742495	24	143788479
14	123710879	*	*	*	*	25	159581692

b=29

e	number	e	number	e	number	e	number
11	140036123	15	127372633	18	131550677	21	131111552
12	138228351	16	122932880	19	124535892	22	132563088
13	130069810	17	132157146	20	127605115	23	143102547
14	125344556	*	*	*	*	24	147638733

b=30

e	number	e	number	e	number	e	number
11	135103844	14	125265674	18	127339399	21	138016411
12	132238323	15	121334436	19	123602536	22	137463547
13	134228784	16	126749166	20	131992174	23	138903985

b=31

e	number	e	number	e	number	e	number
11	136921327	14	127472505	17	127359826	20	140352214
12	130534984	15	126438527	18	128885616	21	138127080
13	136161798	16	123575313	19	133251597	22	139223997

b=32

e	number	e	number	e	number	e	number	e	number
11	144683623	13	127323270	15	130647016	18	134239948	20	140158737
12	134195093	14	127496228	17	132106893	19	129974954	21	149285802

b	e	number	b	e	number	b	e	number
33	11	136276139	33	17	136383758	34	13	146606427
33	12	130762412	33	18	132304052	34	14	136784165
33	13	132180675	33	19	134971056	34	16	139747717
33	14	129776191	33	20	139984910	34	17	144667060
33	15	138300646	34	11	136950958	34	18	142540300
33	16	136476148	34	12	138164882	34	19	143303870

b	e	number	b	e	number	b	e	number
35	11	138669868	35	16	140132722	36	12	143216037
35	12	135005688	35	17	137980825	36	13	139766220
35	13	139912609	35	18	142403698	36	15	140978175
35	14	134366843	36	11	139722400	36	16	145027496
35	15	138141530	*	*	*	36	17	143002581

The number of centers is 152. The total number of the squares with a = 10 is 17 629 237 298

Table 7. A list of the number of four corner magic squares with a = 11

b	e	number	b	e	number	b	e	Number
19	17	110552567	21	17	114893203	22	16	116823725
20	16	114569570	21	18	118305846	22	17	123533289
20	18	114008286	21	19	119948160	22	18	121716664
20	19	117069993	21	20	124970823	22	20	128878094
21	15	118417701	22	14	116963054	22	21	129733782

b	e	number	b	e	number	b	e	number
23	13	124569278	23	21	128935348	24	19	127167523
23	15	120034009	23	22	126422480	24	20	131864055
23	16	118922094	24	14	135746592	24	21	125413385
23	18	120317909	24	16	122087856	24	22	136485771
23	19	123815800	24	17	128636624	24	23	133111798
*	*	*	24	18	121267077	*	*	*

b=25

e	number	e	number	e	number	e	number
14	138638127	17	119720366	20	121651619	22	129562565
15	121148107	18	119842749	21	127385452	23	132874912
16	118988397	*	*	*	*	24	148961118

b=27

e	number	e	number	e	number	e	number
12	132814270	15	120140520	19	120144630	22	122786061
13	125824360	16	118268595	20	120616795	23	136590042
14	119430806	17	118534435	21	126132561	24	143210116

b=28

e	number	e	number	e	number	e	number
12	124919657	15	117656189	18	127670716	21	127833984
13	125273859	16	120791061	19	121782040	22	135534100
14	121918308	17	128224555	20	122293967	23	133820176

b=29

e	number	e	number	e	number	e	number
12	125234347	15	122573260	18	125961832	20	138563569
13	126975785	16	121538002	19	125956072	21	135859567
14	133891538	*	*	*	*	22	133672620

b=30

e	number	e	number	e	number	e	number
12	127953676	15	130114491	17	126564756	19	126912377
13	123151022	16	126297696	18	137071065	20	131085273
14	123922390	*	*	*	*	21	136143199

b=31

e	number	e	number	e	number	e	number
12	124459006	14	123465909	17	124853715	19	130647564
13	124494764	15	120421691	18	126897375	20	131353405

b=32

e	number	e	number	e	number	e	number
12	135893351	14	144373363	16	129171902	18	130923215
13	125992751	15	128140980	17	144748454	19	143686572

b	e	number	b	e	number	b	e	number
33	12	135241642	33	17	129815439	34	14	134816670
33	13	125920989	33	18	141023114	34	15	134200506
33	14	135777936	34	12	131879763	34	16	136351628
33	16	132950667	34	13	134564403	34	17	138147354

b	e	number	b	e	number
35	12	135099351	36	12	137808915
35	13	136316609	36	13	139683145
35	15	139228056	36	14	143478700
35	16	136652741	36	15	139145267

The number of centers is 121. The total number of the squares with a = 11 is 15 244 199 949

Table 8. A list of the number of four corner magic squares with a = 12

b	e	number	b	e	number	b	e	number
19	17	114008286	21	15	116823725	22	16	115808535
20	16	114893203	21	17	124468426	22	17	122966558
20	18	115335036	21	18	121813151	22	19	134911944
20	19	127042593	21	19	132696180	22	21	140287525
23	15	119960980	23	21	127885765	24	18	119495452
23	17	119686743	23	22	128159389	24	20	131864055
23	18	123936730	24	15	118612881	24	21	125413385
23	19	133595891	24	16	115167589	24	22	136485771
23	20	131464775	24	17	122456182	24	23	133111798

b=26

e	number	e	number	e	number	e	number
13	138638127	16	119720366	19	121651619	21	129562565
14	121148107	17	119842749	20	127385452	22	132874912
15	118988397	*	*	*	*	23	148961118

b=27

e	number	e	number	e	number	e	number
13	128669997	16	117984880	18	127101891	20	124160058
14	123688776	17	127046582	19	122693147	21	130931690
15	121250300	*	*	*	*	22	132367143

b=28

e	number	e	number	e	number	e	number
13	122798548	15	126016302	18	125793496	20	127789752
14	120248808	16	125211739	19	128757367	21	128695654

b=29

e	number	e	number	e	number	e	number
13	127562643	15	123198686	17	129976728	19	126099322
14	120598342	16	127120241	18	125702258	20	127475085

b	e	number	b	e	number	b	e	number
30	13	128436653	30	18	129474613	31	15	124416619
30	14	123914008	30	19	131937170	31	16	124270943
30	15	128548879	31	13	131727944	31	17	131514121
30	17	130806349	31	14	124574563	31	18	134554710

b	e	number	b	e	number	b	e	number
32	13	127799492	33	13	133627920	34	13	136755260
32	14	129252970	33	14	131550882	34	15	135424627
32	16	133908073	33	15	133492696	35	13	140458558
32	17	129448376	33	16	131248649	35	14	138679259

The number of centers is 87. The total number of the squares with a = 12 is 11 061 888 729

Table 9. A list of the number of four corner magic squares with a = 13

b	e	number	b	e	number	b	e	number
20	18	124677774	22	18	131610336	23	17	124083989
20	19	138772449	22	19	145055513	23	18	126918622
21	17	121004372	22	20	151703169	23	20	124437241
21	18	129786974	22	21	129903345	23	21	128673928
22	16	129832011	23	16	128077692	23	22	137904767

b	e	number	b	e	number	b	e	number
25	14	118612881	25	22	129837213	26	21	133111798
25	15	115167589	26	14	122087856	27	14	123924646
25	16	122456182	26	15	128636624	27	15	117604982
25	17	119495452	26	16	121267077	27	16	127142446
25	19	121035912	26	17	127167523	27	18	132115832
25	20	130324697	26	18	131864055	27	19	122094637
25	21	124824891	26	19	125413385	27	20	129444584
*	*	*	26	20	136485771	*	*	*

b=28

e	number	e	number	e	number
14	127377699	16	133211322	18	125439935
15	122452951	17	131340801	19	133319772

b	e	number	b	e	number	b	e	number
29	14	123267037	30	14	127726225	31	14	126603828
29	15	120589910	30	15	125962725	31	16	130235289
29	17	122905109	30	16	129304966	32	14	130130936
29	18	130555931	30	17	130766044	32	15	134020039

The number of centers is 55. The total number of the squares with a = 13 is 7 037 768 734

Table 10. A list of the number of four corner magic squares with a = 14

b	e	number	b	e	number	b	e	number
20	18	127552211	22	17	132502909	24	16	124083989
20	19	159355574	22	18	129946878	24	17	126918622
21	17	132348932	22	20	133508481	24	19	124437241
21	19	161648995	22	21	133936126	24	20	128673928
21	20	166955808	24	15	128077692	24	21	137904767

b	e	number	b	e	number	b	e	number
25	15	119686743	26	15	120317909	27	17	130357919
25	16	123936730	26	16	123815800	27	18	127579161
25	17	133595891	26	18	128935348	28	15	123651341
25	18	131464775	26	19	126422480	28	17	128145759
25	19	127885765	27	15	124901563	29	15	127508814
25	20	128159389	27	16	124448115	29	16	129419978

The number of centers is 33. The total number of the squares with a = 14 is 4 328 085 633

The number of centers is 4. The total number of the squares with a = 16 is 618 706 214. The total number of the squares with negative center such that a = 9, …, 16 is 82 105 838 720.

In the case of four corner magic squares with positive center we fix by each run of the code two specific values for a, b and e, which satisfy the following conditions:

$a < e < b, \quad a < 2s – a – b – e,$
$b*e– a*(2s – a – b – e) > 0, \; 8 < a < 18.$

We exclude the squares with symmetric centers and semi symmetric centers in the next list. We list the number for all squares with respect to different values of a, b and e in the following tables: In Tables (13 - 19) we list the squares corresponding to a = 9 (res. 10, …, 16).

Table 11. A list of the number of four corner magic squares with a = 15

b	e	number	b	e	number	b	e	number
20	18	147854836	23	19	133508481	24	19	129903345
21	18	145111927	23	20	133936126	25	16	134911944
21	20	144989484	24	16	131610336	25	18	140287525
23	16	132502909	24	17	145055513	26	16	128878094
23	17	129946878	24	18	151703169	26	17	129733782

The number of centers is 15. The total number of the squares with a = 15 is 2 059 934 349

Table 12. A list of the number of four corner magic squares with a = 16

a	b	e	number	a	b	e	number
16	22	17	145111927	16	23	17	161648995
16	22	19	144989484	16	23	18	166955808

Table 13. A list of the number of four corner magic squares with a = 9

b	e	number	b	e	number	b	e	number
15	14	134121525	18	14	130230377	21	11	135320277
16	13	134603089	18	15	119459984	21	12	131887930
16	14	137809464	19	10	149173141	21	13	127300394
16	15	128899433	19	11	139701915	22	10	137244633
17	12	135810466	19	12	133747474	22	11	133571878
17	13	135108324	19	13	129942329	22	12	133190481
17	14	131063343	19	14	126767780	23	10	143761353
17	15	121418695	20	10	138342068	23	11	131190172
17	16	121120361	20	11	133867159	24	10	134848602
18	11	138190635	20	12	136645500	24	11	136686393
18	12	132200077	20	13	130317255	25	10	132292873
18	13	127926756	21	10	138917234	26	10	141770813

The number of centers is 36. The total number of the squares with a = 9 is 4 804 450 183

Table 14. A list of the number of four corner magic squares with a = 10

b	e	number	b	e	number	b	e	number
15	13	138687369	16	14	131031874	17	13	140330105
15	14	131797562	16	15	130665900	17	14	124066378
16	12	143478320	17	11	137800262	17	15	126281250
16	13	136013377	17	12	132397335	17	16	118375804

b	e	number	b	e	number	b	e	number
18	11	136432511	19	13	123451990	21	13	128316619
18	12	135608506	19	14	122625761	22	11	132569706
18	13	127023594	19	15	118056469	22	12	128229502
18	14	125843075	20	11	134093384	22	13	121958143
18	15	119744242	20	12	132626465	23	11	130730850
18	16	118193688	20	13	134963983	23	12	132054199
19	11	134995272	20	14	119715913	24	11	134847246
19	12	126840267	21	11	142075092	25	11	145534400
*	*	*	21	12	127958340	*	*	*

The number of centers is 37. The total number of the squares with a = 10 is 4 825 414 753

Table 15. A list of the number of four corner magic squares with a = 11

b	e	number	b	e	number	b	e	number
14	13	142088500	17	16	119672738	19	16	114191892
15	12	136855015	18	12	133569930	20	12	126870022
15	13	135246322	18	13	122768564	20	13	120619213
15	14	131447556	18	14	122474344	20	14	130615818
16	12	134760196	18	15	125037058	20	15	110774224
16	13	131115942	18	16	115067660	21	12	126432333
16	14	129648211	18	17	118351247	21	13	125776194
16	15	121839807	19	12	136352497	21	14	121807251
17	12	132847711	19	13	125854607	22	12	123942478
17	13	124754709	19	14	121366787	22	13	123377288
17	14	135611220	19	15	118971662	23	12	126558532
17	15	119458731	*	*	*	24	12	134159325

The number of centers is 35. The total number of the squares with a = 11 is 4 420 285 584

Table 16. A list of the number of four corner magic squares with a = 12

b	e	number	b	e	number	b	e	number
14	13	141617171	17	16	121499468	19	16	113047908
15	13	134264186	18	13	129087754	20	13	125161145
15	14	128764559	18	14	122986578	20	14	114746404
16	13	131439570	18	15	118791505	20	15	117601428
16	14	126599940	18	16	116805362	21	13	122029111
16	15	118468772	18	17	116497767	21	14	118858318
17	13	126275986	19	13	129410433	22	13	125597483
17	14	119401141	19	14	119205291	22	14	120034009
17	15	127438703	19	15	121100417	23	13	135746592

The number of centers is 27. The total number of the squares with a = 12 is 3 342 477 001

Table 17. A list of the number of four corner magic squares with a = 13

b	e	number	b	e	number	b	e	number
15	14	130498831	18	15	119115855	19	17	115335036
16	14	125313274	18	16	121594118	20	14	119898468
16	15	124866991	18	17	117069993	20	15	123533289
17	14	123980424	19	14	124584159	20	16	124468426
17	15	120442628	19	15	114368512	21	14	118922094
17	16	122373478	19	16	118305846	21	15	115808535
18	14	123890374	*	*	*	22	14	119960980

The number of centers is 20. The total number of the squares with a = 13 is 2 424 331 311

Table 18. A list of the number of four corner magic squares with a = 14

b	e	number	b	e	number	b	e	number
16	15	125477566	18	16	119948160	19	17	124677774
17	15	126616800	18	17	127042593	20	15	122966558
17	16	124594623	19	15	121716664	20	16	121004372
18	15	120465231	19	16	121813151	21	15	129832011

The number of centers is 12. The total number of the squares with a = 14 is 1 486 155 503

Table 19. A list of the number of four corner magic squares with a = 15, 16

a	b	e	number	a	b	e	number
15	17	16	124970823	15	20	16	132348932
15	18	16	132696180	15	18	17	138772449
15	19	16	129786974	15	19	17	127552211
16	18	17	159355574	16	19	17	147854836

The number of centers is 8. The total number of the squares with a = 15, 16 is 1 093 337 979. The total number of the squares with

$$a = 9, \ldots, 16$$

is

$$22\ 396\ 452\ 314.$$

When we join both results together we conclude that: the total number of the squares such that

$$a = 9, 10, \ldots, 16$$

is

$$104\ 502\ 291\ 034.$$

Hence, there are

$$104\ 502\ 291\ 034 * 16 = 1\ 672\ 036\ 656\ 544$$

different four corner magic squares of order six with a > 8, which do not have neither a symmetric center nor a semi symmetric center.

3. THE NUMBER OF FOUR CORNER MAGIC SQUARES OF ORDER 6

The number of all different possible values for a, b and e by computing the number of four corner magic squares is 3429. Hence, there are 3429 possible centers of the natural four corner magic squares.

There are 153 possible symmetric centers of the natural four corner magic squares. According to [13] there are

$$28\ 634\ 584\ 244 * 16 = 458\ 153\ 347\ 904$$

different natural four corner magic squares with symmetric center. There are 306 possible semi symmetric centers of the natural four corner magic squares. According to [11] there are

$$101\ 425\ 060\ 998 * 16 = 1\ 622\ 800\ 975\ 968$$

different natural four corner magic squares with semi symmetric center.

Based on the information about the computed natural four corner magic squares in this paper we will estimate the whole number. We have considered

$$153 + 306 + 650 + 175 = 1284$$

centers. The total number of squares associated with them is

$$1\ 672\ 036\ 656\ 544 + 458\ 153\ 347\ 904 + 1\ 622\ 800\ 975\ 968 = 3\ 752\ 990\ 980\ 416.$$

We want here to estimate the number of four corner magic squares of order 6. By computing the average number of squares per center

$$\frac{3\,752\,990\,980\,416}{1284} \cong 2\,922\,890\,172\,.$$

Hence, we estimate the number of the four corner magic squares to be

$$2\,922\,890\,172 * 3429 \cong 10^{13}.$$

4. PARALLELIZATION AND GRID COMPUTING

The problem itself is split into several uncorrelated problems, since counting squares for each center is a separate problem. The code is constructed so that the input is the center of the square. This is the first step by splitting the job of counting into many smaller jobs, which run in parallel. We can fix the value of the outer for-loop before running the code. By this way we can split the task into 36 tasks, which can run in parallel.

The used code is easy parallelizable, where no data exchange between the parallel running tasks is necessary. The code uses nested loops representing the independent variables (which are the small letters in the formula). The first loop is for the variable t. When we fix one center we also run the code for an interval of the values of the variable t. This interval is part of the input. We have the freedom to choose any subinterval of (0,36). By choosing smaller intervals we run smaller jobs since they do not involve much more computations.

5. CONCLUSION

The problem of counting the number of squares of order six has not been completely solved yet. We can find some numbers and estimations in [9]. The development of computers can help by performing these calculations. In this paper we presented some counting results and ideas about how to perform future counting. For detailed C-code, which we use, there is a description in [12].

COMPETING INTERESTS

Author has declared that no competing interests exist.

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