Journal of Advances in Mathematics and Computer Science

35(5): 45-67, 2020; Article no.JAMCS.59314

ISSN: 2456-9968
(Past name: British Journal of Mathematics & Computer Science, Past ISSN: 2231-0851)

A Comparative Study of Solving Methods of Transportation Problem in Linear Programming Problem

Farzana Sultana Rafi[1*] and Safiqul Islam[1]

[1]Department of Applied Mathematics, Noakhali Science and Technology University, Noakhali, 3814, Bangladesh.

Authors' contributions

This work was carried out in collaboration between both authors. Author FSR designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author SI managed the analyses of the study and the literature searches. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/JAMCS/2020/v35i530281
Editor(s):
(1) Dr. Rodica Luca, Gheorghe Asachi Technical University of Iaşi, Romania.
(2) Dr. Zhenkun Huang, Jimei University, China.
(3) Dr. Wei-Shih Du, National Kaohsiung Normal University, Taiwan.
Reviewers:
(1) Rakesh Kumar Bajaj, Jaypee University of Information Technology, India.
(2) Rahul Kar, Springdale High School (H.S), India.
(3) M. Mohamed Jeyavuthin, India.
Complete Peer review History: http://www.sdiarticle4.com/review-history/59314

Opinion Article

Received: 10 May 2020
Accepted: 18 July 2020
Published: 30 July 2020

Abstract

The paper is related with the basic transportation problem (TP)which is one kind of linear programming problem (LPP). There are some existing methods for solving transportation problem and in this paper all the standard existing methods has been discussed to understand which one is the best method among them. Among all of existing methods, the Vogel's Approximation Method (VAM) is considered the best method which gives the better optimal result then other methods and North-West Corner Rule is considered as simplest but gives worst result. A C programming code for Vogel's Approximation Method have been added in the appendix.

Keywords: Linear programming problem; transportation problem; north west corner rule; Vogel's approximation method; optimal solution; basic feasible solution.

**Corresponding author: E-mail: rafi.farzana@yahoo.com;*

1 Introduction

Transportation problem is one of the known methods in operation research for its real-life application [1]. It is playing an important role in our modern life in the case of shipping of goods from sources to destinations and in transporting goods companies expend huge amount of money [2]. Basically, the transportation problems are related with the optimal (best possible) way in which a product produced at different sources (factories or plants) can be transported to a number of different destinations (warehouses or customers) [3]. Transportation problem is the method of obtaining "good" solution which also occur in robotics area with great practical significance [4]. Transportation problem is a processing method of optimization technique which is used by many producers to solve regular problem very frequently [5]. Transportation problem is also used in inventory control system, employment scheduling, personal assignment [6]. Concerning with transportation time there are two types of transportation problem: (i) minimization of 1^{st} transportation time (linear) (ii) minimization of 2^{nd} transportation time (nonlinear) [7]. There are several existing methods to solve transportation problem such as Northwest Corner Rule, Least Cost Method, Vogel's Approximation Method, Row Minimum Method, Column Minimum Method [8]. The general parameters of TP are resources (are goods, machines, tools, people, cargo, and money), Locations (depot, nodes, railway stations, bus stations, loading port, seaports, airports) transportation modes (ship, aircraft, truck, train, pipeline, motorcycle) [9]. In 1939 L. Kantorovich published the first outcome of research in the organization and planning of production. During second world war, F. Hitchcock gives the first mathematical model of transportation model. After this, G. Dantzig represents the transportation problem as a special problem of linear programming problem [10]. The main aim of transportation problem is to minimize total transportation costs by satisfying destination requirements within source requirements [11]. Finding the initial basic feasible solution and then using this find the optimal solution is the primary process of solving transportation problem [12].

2 Preliminaries of Transportation Problem

The Transportation problem is a special class of linear programming problem, which mainly deals with logistics. Transportation problem relates to distribute a product from a number of origins (plants or factories) to a number of demand destinations (warehouses or market). The objective is to satisfy the demands from the supply constraints within the plant's capacity at minimum transportation cost.

2.1 Network representation of transportation problem

A simple network diagram of transportation problem is illustrated in the following Fig. 1.

2.2 Classifications of transportation problem

2.2.1 Balanced transportation problem

A Transportation Problem is said to be balanced Transportation Problem if the total number of supplies is the same as the total number of demands.

2.2.2 Unbalanced transportation problem

A Transportation Problem is said to be unbalanced Transportation Problem if the total number of supplies is not the same as the total number of demands.

In this paper, we only considered the balanced transportation problem.

2.3 Tabular representation of transportation problem

A balanced transportation problem having m sources of supply$s_1, s_2, \ldots\ldots, s_m$with $a_i(i = 1,2,\ldots,m)$ unit of supplies and n destinations $d_1, d_2, \ldots\ldots, d_n$with$b_j(j = 1,2,\ldots,n)$ unit of requirements can be represented in a Table 1 as follows.

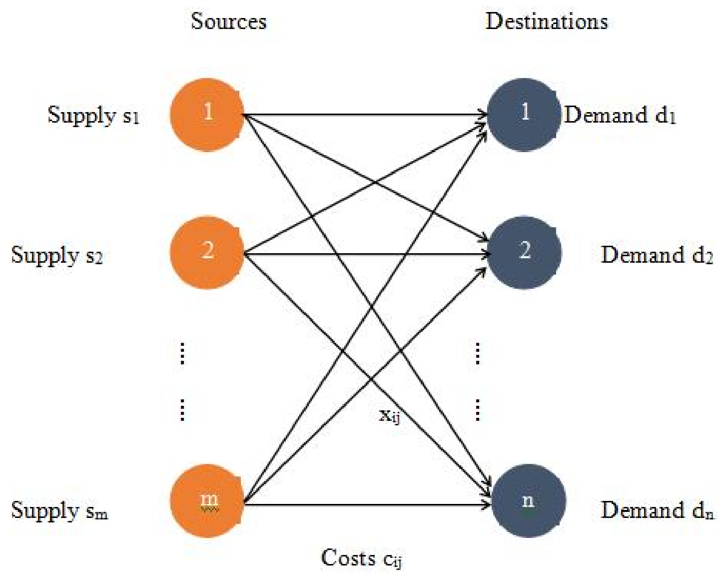

Fig. 1. Network representation of transportation problem

Table 1. Tabular representation of transportation problem

To from	d_1	d_2	……	d_n	Supply (a_i)
s_1	c_{11}	c_{12}	……	c_{1n}	a_1
s_2	c_{21}	c_{22}	……	c_{2n}	a_2
⁞	⁞	⁞	⁞	⁞	⁞
⁞	⁞	⁞	⁞	⁞	⁞
s_m	c_{m1}	c_{m2}	……	c_{mn}	a_m
Demand (b_j)	b_1	b_2	……	b_n	$\sum a_i = \sum b_j$

2.4 Mathematical formulations of transportation problem

Mathematically a transportation problem is nothing but a special linear programming problem in which the objective function is to minimize the cost of transportation subjected to the demand and supply constraints.

It applies to situations where a single commodity is transported from various sources of supply (origins) to various demands (destinations).

Let there be m sources of supply $s_1, s_2, \ldots\ldots, s_m$having $a_i(i = 1,2,\ldots,m)$ units of supplies respectively to be transported among n destinations $d_1, d_2, \ldots\ldots, d_n$with$b_j(j = 1,2,\ldots,n)$ units of requirements respectively.

Let c_{ij} be the cost of shipping one unit of commodity from source i to destination j for each route. If x_{ij} represent the units shipped per route from source i, to destination j, then the problem is to determine the transportation schedule which minimizes the total transportation cost of satisfying supply and demand conditions.

Minimize $z = \sum_{i}^{m} \sum_{j}^{n} c_{ij} x_{ij}$

Subject to the constraints,

$\sum_{j}^{n} x_{ij} = a_i,\ i = 1,2,\dots,m\ (supply\ constraints)$
$\sum_{i}^{m} x_{ij} = b_j,\ j = 1,2,\dots,n\ (demand\ constraints)$
$x_{ij} \geq 0$ for all i and j

3 Established Methods to Find the Solution of Transportation Problem

The models given below are always used for solving the transportation problems.

- North-west Corner Rule (NWC)
- Row Minima Method (RMM)
- Column Minima Method (CMM)
- Least Cost Method (LCM)
- Vogel's Approximation Method (VAM)

3.1 North-west corner rule (NWC)

The so-called Northwest corner rule appears in virtually every text-book chapter on the transportation problem. It is a standard method for computing a basic feasible solution and it does so by fixing the values of the basic variables one by one and starting from the Northwest corner of matrix.

The North-west corner rule is very simple and easy to use and apply. However, it is not sensitive to costs and consequently yields to poor initial solutions. The processing method of North-west Corner Rule is:

Step 1: Select the upper left-hand corner cell of the transportation table and allocate as many units as possible equal to the minimum between available supply and demand, i.e. $min(s_1, d_1)$

Step 2: Adjust the supply and demand numbers in the respective rows and columns.

Step 3: If the demand for the first cell is satisfied, then move horizontally to the next cell in the second column.

Step 4: If the supply for the first row is exhausted, then move down to the first cell in the second row.

Step 5: If for any cell, supply equals demand, then the next allocation can be made in cell either in the next row or column.

Step 6: Continue the process until all supply and demand values are exhausted.

3.1.1 Numerical example 1

Find Solution using North-West Corner Rule

Table 3.1.1(a). Table of example 1

	D1	D2	D3	D4	Supply
S1	11	13	17	14	250
S2	16	18	14	10	300
S3	21	24	13	10	400
Demand	200	225	275	250	

Solution:

The rim values for $S1$=250 and $D1$=200 is compared.

The smaller of the two i.e. min (250,200) = **200** is assigned to $S1$ $D1$.

This meets the complete demand of $D1$ and leaves 250 - 200 = 50 units with $S1$

Table 3.1.1(b). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11**(200)**	13	17	14	50
*S*2	16	18	14	10	300
*S*3	21	24	13	10	400
Demand	0	225	275	250	

The rim values for $S1$=50 and $D2$=225 are compared.

The smaller of the two i.e. min (50,225) = **50** is assigned to $S1$ $D2$.

This exhausts the capacity of $S1$ and leaves 225 - 50 = 175 units with $D2$

Table 3.1.1(c). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11**(200)**	13**(50)**	17	14	0
*S*2	16	18	14	10	300
*S*3	21	24	13	10	400
Demand	0	175	275	250	

The rim values for $S2$=300 and $D2$=175 are compared.

The smaller of the two i.e. min (300,175) = **175** is assigned to $S2$ $D2$

This meets the complete demand of $D2$ and leaves 300 - 175 = 125 units with $S2$

Table 3.1.1(d). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11**(200)**	13**(50)**	17	14	0
*S*2	16	18**(175)**	14	10	125
*S*3	21	24	13	10	400
Demand	0	0	275	250	

The rim values for $S2$=125 and $D3$=275 are compared.

The smaller of the two i.e. min (125,275) = **125** is assigned to *S*2 *D*3.

This exhausts the capacity of *S*2 and leaves 275 - 125 = 150 units with *D*3

Table 3.1.1(e). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11**(200)**	13**(50)**	17	14	0
*S*2	16	18**(175)**	14**(125)**	10	0
*S*3	21	24	13	10	400
Demand	0	0	150	250	

The rim values for *S*3=400 and *D*3=150 are compared.

The smaller of the two i.e. min (400,150) = **150** is assigned to *S*3 *D*3.

This meets the complete demand of *D*3 and leaves 400 - 150 = 250 units with *S*3

Table 3.1.1(f). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11**(200)**	13**(50)**	17	14	0
*S*2	16	18**(175)**	14**(125)**	10	0
*S*3	21	24	13**(150)**	10	250
Demand	0	0	0	250	

The rim values for *S*3=250 and *D*4=250 are compared.

The smaller of the two i.e. min (250,250) = **250** is assigned to *S*3 *D*4.

Table 3.1.1(g). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11**(200)**	13**(50)**	17	14	0
*S*2	16	18**(175)**	14**(125)**	10	0
*S*3	21	24	13**(150)**	10**(250)**	0
Demand	0	0	0	0	

Initial feasible solution is

Table 3.1.1(h). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11 **(200)**	13 **(50)**	17	14	250
*S*2	16	18 **(175)**	14 **(125)**	10	300
*S*3	21	24	13 **(150)**	10 **(250)**	400
Demand	200	225	275	250	

The minimum total transportation cost =11×200+13×50+18×175+14×125+13×150+10×250=12200

3.1.2 Numerical example 2

Find Solution using North-West Corner Rule

Table 3.1.2(a). Solve the following example

To Form	D	E	F	Supply
A	6	4	1	50
B	3	8	7	40
C	4	4	2	60
Demand	20	95	35	150

Solution:

Table 3.1.2(b). Solution of example 2

To Form	D	E	F	Supply
A	20	30	1	0
B	3	40	7	0
C	4	25	35	0
Demand	0	0	0	150

Number of basic variables = m + n – 1 = 3+3-1 = 5

The total transportation cost is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

$$6 \times 20 + 4 \times 30 + 8 \times 40 + 4 \times 25 + 2 \times 35 = 730$$

3.1.3 Numerical example 3

Find Solution using North-West Corner Rule

Table 3.1.3(a). Solve the following example

To From	A	B	C	D	E	Supply
P	4	1	3	4	4	60
Q	2	3	2	2	3	35
R	3	5	2	4	4	40
Demand	22	45	20	18	30	135

Solution:

Table 3.1.3(b). Solution of example 3

To From	A	B	C	D	E	Supply
P	22	38	3	4	4	0
Q	2	7	20	8	3	0
R	3	5	2	10	30	0
Demand	0	0	0	0	0	135

Number of basic variables = m + n – 1 = 5+3-1=7

The total transportation cost is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

$4 \times 22 + 1 \times 38 + 3 \times 7 + 2 \times 20 + 2 \times 8 + 4 \times 10 + 4 \times 30 = 363$

3.2 Row minima method (RMM)

In this method we allocate maximum possible in the lowest cost cell of the first row. The idea is to exhaust either the capacity of the first source or the demand at destination center is satisfied or both. Continue the process for the other reduced transportation costs until all the supply and demand conditions are satisfied. The minimum transportation cost can be obtained by following the steps given below:

Step 1: In this method, we allocate as much as possible in the lowest cost cell of the first row, i.e. allocate $min(s_i, d_j)$.

Step 2: a. Subtract this min value from supply $s_i$and demand d_jb. If the supply s_i is 0, then cross (strike) that row and if the demand d_j is 0 then cross (strike) that column. c. If min unit cost cell is not unique, then select the cell where maximum allocation can be possible Step 3: Repeat this process for all uncrossed rows and columns until all supply and demand values are 0.

3.2.1 Numerical example 1

Find Solution using Row Minima Method

Table 3.2.1.(a). Table of example 1

	D1	D2	D3	D4	Supply
S1	11	13	17	14	250
S2	16	18	14	10	300
S3	21	24	13	10	400
Demand	200	225	275	250	

Solution:

In 1st row, the smallest transportation cost is 11 in cell *S*1*D*1.

The allocation to this cell is min (250,200) = **200**.

This satisfies the entire demand of *D*1 and leaves 250 - 200 = 50 units with *S*1

Table 3.2.1(b). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11**(200)**	13	17	14	50
*S*2	16	18	14	10	300
*S*3	21	24	13	10	400
Demand	0	225	275	250	

In 1st row, the smallest transportation cost is 13 in cell S1D2.

The allocation to this cell is min (50,225) = **50**.

This exhausts the capacity of S1 and leaves 225 - 50 = 175 units with D2.

Table 3.2.1(c). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11**(200)**	13**(50)**	17	14	0
*S*2	16	18	14	10	300
*S*3	21	24	13	10	400
Demand	0	175	275	250	

In 2nd row, the smallest transportation cost is 10 in cell S2D4.

The allocation to this cell is min (300,250) = **250**.

This satisfies the entire demand of D4 and leaves 300 - 250 = 50 units with S2

Table 3.2.1(d). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11**(200)**	13**(50)**	17	14	0
*S*2	16	18	14	10**(250)**	50
*S*3	21	24	13	10	400
Demand	0	175	275	0	

In 2nd row, the smallest transportation cost is 14 in cell S2D3.

The allocation to this cell is min (50,275) = **50**.

This exhausts the capacity of S2 and leaves 275 - 50 = 225 units with D3

Table 3.2.1(e). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11**(200)**	13**(50)**	17	14	0
*S*2	16	18	14**(50)**	10**(250)**	0
*S*3	21	24	13	10	400
Demand	0	175	225	0	

In 3rd row, the smallest transportation cost is 13 in cell S3D3.

The allocation to this cell is min (400,225) = **225**.

This satisfies the entire demand of D3 and leaves 400 - 225 = 175 units with S3.

Table 3.2.1(f). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11**(200)**	13**(50)**	17	14	0
*S*2	16	18	14**(50)**	10**(250)**	0
*S*3	21	24	13**(225)**	10	175
Demand	0	175	0	0	

In 3rd row, the smallest transportation cost is 24 in cell S3D2.

The allocation to this cell is min (175,175) = **175**.

Table 3.2.1(g). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11**(200)**	13**(50)**	17	14	0
*S*2	16	18	14**(50)**	10**(250)**	0
*S*3	21	24**(175)**	13**(225)**	10	0
Demand	0	0	0	0	

Initial feasible solution is

Table 3.2.1(h). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11 **(200)**	13 **(50)**	17	14	250
*S*2	16	18	14 **(50)**	10 **(250)**	300
*S*3	21	24 **(175)**	13 **(225)**	10	400
Demand	200	225	275	250	

The minimum total transportation cost =11×200+13×50+14×50+10×250+24×175+13×225=13175

3.2.2 Numerical example 2

Find Solution using Row Minima Method

Table 3.2.2(a). Solve the following example

To **Form**	**D**	**E**	**F**	**Supply**
A	6	4	1	50
B	3	8	7	40
C	4	4	2	60
Demand	20	95	35	150

Solution:

Table 3.2.2(b). Solution of example 2

To **Form**	**D**	**E**	**F**	**Supply**
A	6	15	35	0
B	20	20	7	0
C	4	60	2	0
Demand	0	0	0	150

Number of basic variables = m + n – 1 = 3+3-1 = 05

The total transportation cost is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

4×15+1×35+3×20+8×20+4×60=555

3.3 Column Minima Method (CMM)

In this method, we start with the first column and allocate as much as possible in the lowest cost cell of column, so that either the demand of the first destination center is satisfied or the capacity of the 2nd is exhausted or both. The minimum transportation cost can be obtained by following the steps given below

Step 1: In this method, we allocate as much as possible in the lowest cost cell of the first Column, i.e. allocate$min(s_i, d_j)$.

Step 2: a. Subtract this min value from supply s_i and demand d_j. b. If the supply s_i is 0, then cross (strike) that row and If the demand $d_j$is 0 then cross (strike) that column. c. If min unit cost cell is not unique, then select the cell where maximum allocation can be possible

Step3: Repeat this process for all uncrossed rows and columns until all supply and demand values are 0.

3.3.1 Numerical example 1

Find Solution using Column minima method

Table 3.3.1.(a). Table of example 1

	D1	**D2**	**D3**	**D4**	**Supply**
S1	11	13	17	14	250
S2	16	18	14	10	300
S3	21	24	13	10	400
Demand	200	225	275	250	

Solution:

In $1st$ column, the smallest transportation cost is 11 in cell $S1D1$

The allocation to this cell is min (250,200) = **200**.

This satisfies the entire demand of $D1$ and leaves 250 - 200 = 50 units with $S1$

Table 3.3.1.(b). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11**(200)**	13	17	14	50
*S*2	16	18	14	10	300
*S*3	21	24	13	10	400
Demand	0	225	275	250	

In 2nd column, the smallest transportation cost is 13 in cell S1D2

The allocation to this cell is min (50,225) = **50**.

This exhausts the capacity of S1 and leaves 225 - 50 = 175 units with D2

Table 3.3.1.(c). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11**(200)**	13**(50)**	17	14	0
*S*2	16	18	14	10	300
*S*3	21	24	13	10	400
Demand	0	175	275	250	

In 2nd column, the smallest transportation cost is 18 in cell S2D2

The allocation to this cell is min (300,175) = **175**.

This satisfies the entire demand of D2 and leaves 300 - 175 = 125 units with S2.

Table 3.3.1(d). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11**(200)**	13**(50)**	17	14	0
*S*2	16	18**(175)**	14	10	125
*S*3	21	24	13	10	400
Demand	0	0	275	250	

In 3rd column, the smallest transportation cost is 13 in cell S3D3

The allocation to this cell is min (400,275) = **275**.

This satisfies the entire demand of D3 and leaves 400 - 275 = 125 units with S3

Table 3.3.1(e). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11**(200)**	13**(50)**	17	14	0
*S*2	16	18**(175)**	14	10	125
*S*3	21	24	13**(275)**	10	125
Demand	0	0	0	250	

In 4th column, the smallest transportation cost is 10 in cell S2D4

The allocation to this cell is min (125,250) = **125**.

This exhausts the capacity of S2 and leaves 250 - 125 = 125 units with D4

Table 3.3.1(f). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11**(200)**	13**(50)**	17	14	0
*S*2	16	18**(175)**	14	10**(125)**	0
*S*3	21	24	13**(275)**	10	125
Demand	0	0	0	125	

In 4th column, the smallest transportation cost is 10 in cell S3D4

The allocation to this cell is min (125,125) = **125**.

Table 3.3.1(g). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11**(200)**	13**(50)**	17	14	0
*S*2	16	18**(175)**	14	10**(125)**	0
*S*3	21	24	13**(275)**	10**(125)**	0
Demand	0	0	0	0	

Initial feasible solution is

Table 3.3.1(h). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11 **(200)**	13 **(50)**	17	14	250
*S*2	16	18 **(175)**	14	10 **(125)**	300
*S*3	21	24	13 **(275)**	10 **(125)**	400
Demand	200	225	275	250	

The minimum total transportation cost =11×200+13×50+18×175+10×125+13×275+10×125=12075

3.3.2 Numerical example 2

Find Solution using Column minima method

To Form	**D**	**E**	**F**	**Supply**
A	6	4	1	50
B	3	8	7	40
C	4	4	2	60
Demand	20	95	35	150

Solution:

Table 3.3.2 Solution of example 2

To Form	**D**	**E**	**F**	**Supply**
A	6	35	15	0
B	20	8	20	0
C	4	60	2	0
Demand	0	0	0	150

Number of basic variables = m + n – 1 = 3+3-1 = 05

The total transportation cost is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

4×35+1×15+3×20+7×20+4×60=595

3.4 Least Cost Method (LCM)

The Least Cost Method is another method used to obtain the initial feasible solution for the transportation problem. Here, the allocation begins with the cell which has the minimum cost. The lower cost cells are chosen over the higher-cost cell with the objective to have the least cost of transportation. The Least Cost Method is considered to produce more optimal results than the North-west Corner because it considers the shipping cost while making the allocation, whereas the North-West corner method only considers the availability and supply requirement and allocation begin with the extreme left corner, irrespective of the shipping cost. The minimum transportation cost can be obtained by following the steps given below:

Step 1: Select the cell having minimum unit cost c_{ij} and allocate as much as possible, i.e. $min\ (s_i, d_j)$

Step 2: a. Subtract this min value from supply s_i and demandd_j.

b. If the supply $s_i$is 0, then cross (strike) that row and if the demand d_j is 0 then cross (strike) that column.

c. If min unit cost cell is not unique, then select the cell where maximum allocation can be possible.

Step 3: Repeat these steps for all uncrossed rows and columns until all supply and demand values are 0.

3.4.1 Numerical example 1

Find Solution using Least Cost Method

Table 3.4.1(a). Transportation table of example 1

	D1	D2	D3	D4	Supply
S1	11	13	17	14	250
S2	16	18	14	10	300
S3	21	24	13	10	400
Demand	200	225	275	250	

Solution:

The smallest transportation cost is 10 in cell $S3D4$

The allocation to this cell is min (400,250) = **250**.

This satisfies the entire demand of $D4$ and leaves 400 - 250 = 150 units with $S3$

Table 3.4.1(b). Solution table of example 1

	*D*1	*D*2	*D*3	*D*4	Supply
*S*1	11	13	17	14	250
*S*2	16	18	14	10	300
*S*3	21	24	13	10**(250)**	150
Demand	200	225	275	0	

The smallest transportation cost is 11 in cell $S1D1$

The allocation to this cell is min (250,200) = **200**.

This satisfies the entire demand of $D1$ and leaves 250 - 200 = 50 units with $S1$

Table 3.4.1(b). Solution table of example 1

	*D*1	*D*2	*D*3	*D*4	Supply
*S*1	11**(200)**	13	17	14	50
*S*2	16	18	14	10	300
*S*3	21	24	13	10**(250)**	150
Demand	0	225	275	0	

The smallest transportation cost is 13 in cell $S3D3$

The allocation to this cell is min (150,275) = **150**.

This exhausts the capacity of $S3$ and leaves 275 - 150 = 125 units with $D3$

Table 3.4.1(c). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11**(200)**	13	17	14	50
*S*2	16	18	14	10	300
*S*3	21	24	13**(150)**	10**(250)**	0
Demand	0	225	125	0	

The smallest transportation cost is 13 in cell *S*1*D*2

The allocation to this cell is min (50,225) = **50**.

This exhausts the capacity of *S*1 and leaves 225 - 50 = 175 units with *D*2

Table 3.4.1(d). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11**(200)**	13**(50)**	17	14	0
*S*2	16	18	14	10	300
*S*3	21	24	13**(150)**	10**(250)**	0
Demand	0	175	125	0	

The smallest transportation cost is 14 in cell *S*2*D*3

The allocation to this cell is min(300,125) = **125**.

This satisfies the entire demand of *D*3 and leaves 300 - 125 = 175 units with *S*2

Table 3.4.1(e). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11**(200)**	13**(50)**	17	14	0
*S*2	16	18	14**(125)**	10	175
*S*3	21	24	13**(150)**	10**(250)**	0
Demand	0	175	0	0	

The smallest transportation cost is 18 in cell *S*2*D*2

The allocation to this cell is min (175,175) = **175**.

Table 3.4.1(f). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11**(200)**	13**(50)**	17	14	0
*S*2	16	18**(175)**	14**(125)**	10	0
*S*3	21	24	13**(150)**	10**(250)**	0
Demand	0	0	0	0	

Initial feasible solution is

Table 3.4.1(g). Solution table of example 1

	***D*1**	***D*2**	***D*3**	***D*4**	**Supply**
*S*1	11 **(200)**	13 **(50)**	17	14	250
*S*2	16	18 **(175)**	14 **(125)**	10	300
*S*3	21	24	13 **(150)**	10 **(250)**	400
Demand	200	225	275	250	

The minimum total transportation cost =11×200+13×50+18×175+14×125+13×150+10×250=12200

3.4.2 Numerical example 2

Find Solution using Least Cost Method

Table 3.4.2(a). Solve the following example

To Form	**D**	**E**	**F**	**Supply**
A	6	4	1	50
B	3	8	7	40
C	4	4	2	60
Demand	20	95	35	150

Solution:

Table 3.4.2(b). Solution of example 2

To Form	**D**	**E**	**F**	**Supply**
A	6	15	35	0
B	20	20	7	0
C	4	60	2	0
Demand	0	0	0	150

Number of basic variables = m + n – 1 = 3+3-1 = 05

The total transportation cost is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

4×15+1×35+3×20+8×20+4×60=555

3.5 Vogel's Approximation Method (VAM)

In Vogel's Approximation Method shipping cost is taken into consideration. The minimum transportation cost can be obtained by following the steps given below:

Step 1: Find the cells having smallest and next to smallest cost in each row and write the difference (called penalty) along the side of the table in row penalty.

Step 2: Find the cells having smallest and next to smallest cost in each column and write the difference (called penalty) along the side of the table in each column penalty.

Step 3: Select the row or column with the maximum penalty and find cell that has least cost in selected row or column. Allocate as much as possible in this cell. If there is a tie in the values of penalties then select the cell where maximum allocation can be possible

Step 4: Adjust the supply & demand and cross out (strike out) the satisfied row or column.

Step 5: Repeat these steps until all supply and demand values are 0.

3.5.1 Numerical example 1

Find Solution using Vogel's Approximation Method

Table 3.5.1(a). Table of example 1

	D1	D2	D3	D4	Supply
S1	11	13	17	14	250
S2	16	18	14	10	300
S3	21	24	13	10	400
Demand	200	225	275	250	

Solution:

Table 3.5.1(b). Solution table of example 1

	*D*1	*D*2	*D*3	*D*4	Supply	Row Penalty
*S*1	11	13	17	14	250	2=13-11
*S*2	16	18	14	10	300	4=14-10
*S*3	21	24	13	10	400	3=13-10
Demand	200	225	275	250		
Column penalty	5=16-11	5=18-13	1=14-13	0=10-10		

The maximum penalty, 5, occurs in column D1.

The minimum cij in this column is c11 = 11.

The maximum allocation in this cell is min(250,200) = **200**.

It satisfy demand of D1 and adjust the supply of S1 from 250 to 50 (250 - 200 = 50)

Table 3.5.1(b). Solution table of example 1

	*D*1	*D*2	*D*3	*D*4	Supply	Row Penalty
*S*1	11**(200)**	13	17	14	50	1=14-13
*S*2	16	18	14	10	300	4=14-10
*S*3	21	24	13	10	400	3=13-10
Demand	0	225	275	250		
Column Penalty	--	5=18-13	1=14-13	0=10-10		

The maximum penalty, 5, occurs in column D2.

The minimum cij in this column is c12 = 13.

The maximum allocation in this cell is min(50,225) = 50.

It satisfy supply of S1 and adjust the demand of D2 from 225 to 175 (225 - 50 = 175)

Table 3.5.1(c). Solution table of example 1

	*D*1	*D*2	*D*3	*D*4	Supply	Row Penalty
*S*1	11**(200)**	13**(50)**	17	14	0	--
*S*2	16	18	14	10	300	4=14-10
*S*3	21	24	13	10	400	3=13-10
Demand	0	175	275	250		
Column Penalty	--	6=24-18	1=14-13	0=10-10		

The maximum penalty, 6, occurs in column D2.

The minimum cij in this column is c22 = 18.

The maximum allocation in this cell is min(300,175) = **175**.

It satisfy demand of D2 and adjust the supply of S2 from 300 to 125 (300 - 175 = 125)

Table 3.5.1(d). Solution table of example 1

	*D*1	*D*2	*D*3	*D*4	Supply	Row Penalty
*S*1	11**(200)**	13**(50)**	17	14	0	--
*S*2	16	18**(175)**	14	10	125	4=14-10
*S*3	21	24	13	10	400	3=13-10
Demand	0	0	275	250		
Column Penalty	--	--	1=14-13	0=10-10		

The maximum penalty, 4, occurs in row S2.

The minimum cij in this row is c24 = 10.

The maximum allocation in this cell is min(125,250) = **125**.

It satisfy supply of S2 and adjust the demand of D4 from 250 to 125 (250 - 125 = 125)

Table 3.5.1(e). Solution table of example 1

	*D*1	*D*2	*D*3	*D*4	Supply	Row Penalty
*S*1	11**(200)**	13**(50)**	17	14	0	--
*S*2	16	18**(175)**	14	10**(125)**	0	--
*S*3	21	24	13	10	400	3=13-10
Demand	0	0	275	125		
Column Penalty	--	--	13	10		

The maximum penalty, 13, occurs in column D3.

The minimum cij in this column is c33 = 13.

The maximum allocation in this cell is min(400,275) = **275**.

It satisfy demand of D3 and adjust the supply of S3 from 400 to 125 (400 - 275 = 125)

Table 3.5.1(f). Solution table of example 1

	*D*1	*D*2	*D*3	*D*4	Supply	Row Penalty
*S*1	11**(200)**	13**(50)**	17	14	0	--
*S*2	16	18**(175)**	14	10**(125)**	0	--
*S*3	21	24	13**(275)**	10	125	10
Demand	0	0	0	125		
Column Penalty	--	--	--	10		

The maximum penalty, 10, occurs in row S3.

The minimum cij in this row is c34 = 10.

The maximum allocation in this cell is min(125,125) = **125**.

It satisfy supply of S3 and demand of D4

Initial feasible solution is

Table 3.5.1(h). Solution table of example 1

	*D*1	*D*2	*D*3	*D*4	Supply	Row penalty
*S*1	11**(200)**	13**(50)**	17	14	250	2 | 1 | -- | -- | -- | -- |
*S*2	16	18**(175)**	14	10**(125)**	300	4 | 4 | 4 | 4 | -- | -- |
*S*3	21	24	13**(275)**	10**(125)**	400	3 | 3 | 3 | 3 | 3 | 10 |
Demand	200	225	275	250		
Column	5	5	1	0		
Penalty	--	5	1	0		
	--	6	1	0		
	--	--	1	0		
	--	--	13	10		
	--	--	--	10		

The minimum total transportation cost =11×200+13×50+18×175+10×125+13×275+10×125=12075

3.5.2 Numerical example 2

Find Solution using Vogel's Approximation Method

To Form	D	E	F	Supply
A	6	4	1	50
B	3	8	7	40
C	4	4	2	60
Demand	20	95	35	150

Solution:

Table 3.5.2. Transportation table of example 2

To Form	D	E	F	Supply	Row Penalty
A	6	15	35	50	3 | 3 | 4 | 4 | 4 |
B	20	20	7	40	4 | 1 | 8 | -- | -- |
C	4	60	2	60	2 | 2 | 4 | 4 | -- |
Demand	20	95	35	150	
Column	1	0	1		

To Form	D	E	F	Supply	Row Penalty
Penalty	--	0	1		
	--	0	--		
	--	0	--		
	--	4	--		

Number of basic variables = m + n – 1 = 3+3-1 = 05

The total transportation cost is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

4×15+1×35+3×20+8×20+4×60=555

3 Result and Discussion

From above discussion we can see that among all of existing methods Vogel's Approximation method provides comparatively a better initial basic feasible solution which is either optimal or near optimal solution even though Vogel's Approximation Method takes many more calculations to find an initial solution. We also observed that North-West Corner Rule provides the worst optimal result compering with others existing methods but the method is very simple to understand. The following table contains the comparison of optimal result of the existing methods:

	NWC	RMM	CMM	LCM	VAM
Example 1	12200	13175	12075	12200	12075
Example 2	730	555	595	555	555

4 Conclusion

In this paper we have discussed about transportation problem and existing methods of solving transportation problem with some numerical example and we also compare the results to find the optimal one. From this study we can conclude that among existing methods North-West Corner Rule is simple but gives worst result compare to others. On the other hand, Vogel's Approximation Method contains a long algorithm but provides the best optimal result compare to others. Till now many alternative methods have been proposed for solving transportation problem which can give more better result compared with existing methods.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Ahmed MM, Khan AR, Uddin MS, Ahmed F. A new approach to solve transportation problems. Open J. Optim. 2016;5(1):22–30.
DOI: 10.4236/ojop.2016.51003

[2] Juman ZAMS, Nawarathne NGSA. An efficient alternative approach to solve a transportation problem. Ceylon J. Sci. 2019;48(1):19.
DOI: 10.4038/cjs.v48i1.7584

[3] Asase A. The Transportation problem: Case study of Guiness Ghana Limited. 2011;1–104.

[4] Dharma S, Manan A, Ahmad B. Optimization of Transportation problem with computer aided linear programming 4. Linear Programming (LP) and. 2005;140–144.

[5] Patel RG, Patel BS, Bhathawala PH. On optimal solution of a transportation problem. 2017;13(9): 6201–6208.

[6] Taha HA. Pesquisa operaciona: Uma visão geral. 2008;359.

[7] Nikolić I. Total time minimizing transportation problem. Yugosl. J. Oper. Res. 2007;17(1):125–133. DOI: 10.2298/YJOR0701125N

[8] Hanif M, Rafi FS. A New method for optimal solutions of transportation problems in LPP. J. Math. Res. 2018;10(5):60.
DOI: 10.5539/jmr.v10n5p60

[9] Díaz-Parra O, Ruiz-Vanoye JA, Bernábe Loranca B, Fuentes-Penna A, Barrera-Cámara RA. A survey of transportation problems. J. Appl. Math. 2014.
DOI: 10.1155/2014/848129

[10] Stojanović V, Spalević L, Božinović M. Software application for solving the transportation problem; 2014.
DOI: 10.13140/2.1.1044.6084

[11] Chaudhuri A, De K. A comparative study of transportation problem under probabilistic and fuzzy uncertainties; 2013.

[12] Palaniyappa R. A study on north east corner method in transportation problem and using of object oriented programming model. 2016;98:42639–42641.

Appendix

C Programming Code for VAM

```
#include <stdio.h>
#include <limits.h>

#define TRUE 1
#define FALSE 0
#define N_ROWS 4
#define N_COLS 5

typedefint bool;

int supply[N_ROWS]={50,60,50,50};
int demand[N_COLS]={30,20,70,30,60};

int costs[N_ROWS][N_COLS]={
{16,16,13,22,17},
{14,14,13,19,15},
{19,19,20,23,50},
{50,12,50,15,11}
};
```

```
bool row_done[N_ROWS]={ FALSE };
bool col_done[N_COLS]={ FALSE };

void diff(int j,int len, bool is_row,int res[3]){
int i, c, min1 = INT_MAX, min2 = min1, min_p =-1;
for(i =0; i < len;++i){
if((is_row)? col_done[i]: row_done[i])continue;
       c =(is_row)? costs[j][i]: costs[i][j];
if(c < min1){
          min2 = min1;
          min1 = c;
          min_p = i;
}
elseif(c < min2) min2 = c;
}
   res[0]= min2 - min1; res[1]= min1; res[2]= min_p;
}

void max_penalty(int len1,int len2, bool is_row,int res[4]){
int i, pc =-1, pm =-1, mc =-1, md = INT_MIN;
int res2[3];

for(i =0; i < len1;++i){
if((is_row)? row_done[i]: col_done[i])continue;
       diff(i, len2, is_row, res2);
if(res2[0]> md){
          md = res2[0];/* max diff */
          pm = i;/* pos of max diff */
          mc = res2[1];/* min cost */
          pc = res2[2];/* pos of min cost */
}
}

if(is_row){
       res[0]= pm; res[1]= pc;
}
else{
       res[0]= pc; res[1]= pm;
}
   res[2]= mc; res[3]= md;
}

void next_cell(int res[4]){
int i, res1[4], res2[4];
   max_penalty(N_ROWS, N_COLS, TRUE, res1);
   max_penalty(N_COLS, N_ROWS, FALSE, res2);

if(res1[3]== res2[3]){
if(res1[2]< res2[2])
for(i =0; i <4;++i) res[i]= res1[i];
else
for(i =0; i <4;++i) res[i]= res2[i];
return;
```

```
}
if(res1[3]> res2[3])
for(i =0; i <4;++i) res[i]= res2[i];
else
for(i =0; i <4;++i) res[i]= res1[i];
}

int main(){
int i, j, r, c, q, supply_left =0, total_cost =0, cell[4];
int results[N_ROWS][N_COLS]={0};

for(i =0; i < N_ROWS;++i) supply_left += supply[i];
while(supply_left >0){
    next_cell(cell);
    r = cell[0];
    c = cell[1];
    q =(demand[c]<= supply[r])? demand[c]: supply[r];
    demand[c]-= q;
if(!demand[c]) col_done[c]= TRUE;
    supply[r]-= q;
if(!supply[r]) row_done[r]= TRUE;
    results[r][c]= q;
    supply_left -= q;
    total_cost += q * costs[r][c];
}

printf("    A   B   C   D   E\n");
for(i =0; i < N_ROWS;++i){
printf("%c",'W'+ i);
for(j =0; j < N_COLS;++j)printf("  %2d", results[i][j]);
printf("\n");
}
printf("\nTotal cost = %d\n", total_cost);
return0;
}
```


Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://www.sdiarticle4.com/review-history/59314